УДК 624.04:531.391.3

DEPENDENCE OF THE NATURAL OSCILLATION FREQUENCY OF THE HALF-TILT CONSOLE ON THE NUMBER OF PANELS

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We consider a planar cantilever truss, the mass of which is distributed over the nodes. The solution of the problem using the induction method is found in the Maple computer mathematics system. To determine the forces in the rods, the method of cutting out the nodes is used. The main frequency of oscillation is determined by the method of Dunkerley. The error of the obtained dependence is compared with the numerical solution of the problem of the oscillation of the cargo system and turns out to be very small, decreasing to 1% with an increase in the number of panels.

Keywords: console, natural oscillation frequency, Maple, induction, number of panels

Statement of the problem

Numerical calculations controlled by some analytical estimates acquire greater reliability, especially in cases where the complexity of the design model may inevitably lead to the accumulation of rounding errors. Obtaining analytical estimates that contain not only loads, dimensions, and properties of materials as problem parameters, but also integer parameters that determine the proportions of the structure, became possible with the use of symbolic mathematics systems and the development of the induction method. This method is used to find formulas for the deflection of some flat trusses [1-4] and the oscillation frequency [5,6]. In this paper, we consider a cantilever truss with masses distributed over the nodes (Fig.1).

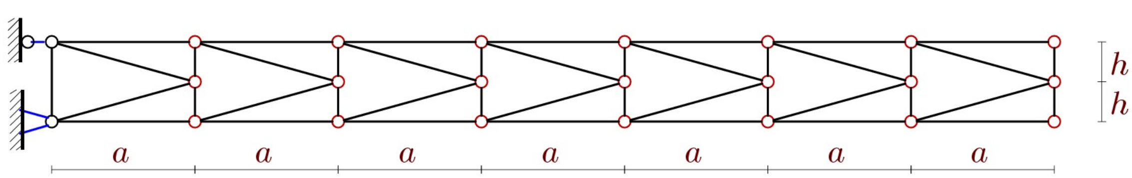

Fig. 1. The truss, $n = 7$

Dunkerley's approximation

To derive the required dependence in an analytical form, we will use the approximate lower approximation of Dunkerley [7]. The number of all nodes of the truss (except for the fixed hinge) endowed with mass is equal to $N = 3n + 1$, the number of rods, including the support rods, $K = 6n + 4$. Dunkerley's formula is as follows:

$$\omega_D{}^2 = 1 / \sum_{i=1}^{N} 1 / \omega_i^2 \qquad (1)$$

where ω_i, $i=1,2,...,3n+1$ is the frequency of the mass m_i oscillation in the absence of all other masses (partial frequency). We write down the differential equation of vertical oscillations of one mass placed in the truss node (the other masses are absent):

$$m\ddot{y}_i + d_{i,n} y_i = 0,$$

where y_i — is the vertical displacement of the mass, $d_{i,n}$ is the stiffness coefficient (i — mass number, n — number of panels). The oscillation frequency of the load

$$\omega_i = \sqrt{d_{i,n} / m} = \sqrt{1/(\delta_{i,n} m)}\,. \tag{2}$$

The compliance coefficient $\delta_{i,n}$ is calculated in terms of the stiffness coefficient and is determined by the Maxwell-Mohr's formula:

$$\delta_{i,n} = 1/d_{i,n} = \sum_{k=1}^{K-3} \left(S_k^i\right)^2 l_k / \left(EF\right). \tag{3}$$

Summation is carried out over all deformable bars of the structure, except for three rigid support bars. To determine the efforts in analytical form, the program [1-4] is used, written in the language of the Maple computer mathematics system. The coordinates of the nodes of the truss are entered into the program. Here is the corresponding fragment of the program:

```
for i to n+1 do  x[i]:=a*i-a;       y[i]:=0;x[i+n+1]:=a*i-a;   y[i+n+1]:=2*h; end:
for i to n do     x[i+2*n+2]:=a*i;  y[i+2*n+2]:=h; end:
```

The lattice structure is specified by conditional vectors containing the numbers of the ends of the corresponding bars:

```
for i to n do N[i]:=[i,i+1];                 N[i+n]:=[i+n+1,i+n+2];
              N[i+2*n]:=[i,i+2*n+2];    N[i+3*n]:=[i+n+1,i+2*n+2];
              N[i+4*n]:=[i+1,i+2*n+2];N[i+5*n]:=[i+n+2,i+2*n+2];end:
N[6*n+1]:=[1,n+2]:
```

Based on these data, a matrix of the system of equations for the equilibrium of nodes is constructed, which consists of the direction cosines of the forces applied to the nodes. External forces (in this case, these are unit forces applied to the nodes containing masses) are taken into account in the vector of the right side of this system. The solution of the system of equations by the inverse matrix method gives the values of the forces included in the Maxwell – Mohr's formula (3).

Placing the mass alternately in the console nodes $i=1,2,...,3n+1$, we obtain in each case the same expression

$$\delta_{i,n} = \left(C_1 a^3 + C_2 c^3 + C_3 h^3\right) / \left(EFh^2\right), \tag{4}$$

where $c=\sqrt{a^2+h^2}$. The coefficients $C_1(n)$.....$C_3(n)$ in this formula are determined by induction. First, the sequences of the coefficients obtained from the solution of the problem for individual trusses are written out, and their common members are found. The length of such sequences should be sufficient to determine the common term. The special operator *rgf_findrecur* of the Maple system gives a recurrence equation from which the desired dependence can be found. The recurrent equation has the form

$$C_{1,n} = 5C_{1,n-1} - 10C_{1,n-2} + 10C_{1,n-3} - 5C_{1,n-4} + C_{1,n-5}$$

Its solution is given by the *rsolve* operator

$$C_1 = n(n-1)(3n^2 - n + 2)/24, \tag{5}$$

Other coefficients are found in the same way:

$$C_2 = 4(8 + 3n^2 + 9n^2),\ C_3 = n(3n+1)/4. \tag{6}$$

Hence, taking into account (1) and (2), we obtain the final formula for the lower bound of the first natural frequency of vibrations of the truss:

$$\omega_D = h\sqrt{EF/(C_1a^3 + C_2c^3 + C_3h^3)}. \tag{7}$$

Formula (6) with coefficients (7), (8) gives a solution to the problem — the expression of the lower bound ω_D depending on the number of panels and the size of the truss.

Numerical calculation of the natural frequency spectrum of the console

We check the obtained solution by comparing it with the lowest frequency of natural oscillations of the system *N* masses. The dynamic equations for a system of massive loads are written in matrix form:

$$\mathbf{M}_N\ddot{\mathbf{Y}} + \mathbf{D_N}\mathbf{Y} = 0, \tag{8}$$

where $\mathbf{Y} = [y_1, y_2, ..., y_N]^T$ is the vector of vertical displacements of masses 1,..., *N,* $\mathbf{M}_N = m\mathbf{I}_N$ — is the inertia matrix of size, $\mathbf{D}_N$ is the stiffness matrix, $\ddot{\mathbf{Y}}$ is the acceleration vector. The pliability matrix $\mathbf{B}_N$, the inverse of the stiffness matrix $\mathbf{D}_N$, is defined using the Mohr's integral

$$b_{i.j} = \sum_{\alpha=1}^{K-3} S_\alpha^{(i)} S_\alpha^{(j)} l_\alpha /(EF) \tag{9}$$

where $S_\alpha^{(i)}$ is the force in the rod α from the action of a single vertical force applied to the node *i*, l_α — is the length of the rod α, EF is the stiffness of the rods. In the number, multiplying equation (2) on the left by the malleability matrix $\mathbf{B}_N$ gives the equation:

$$m\mathbf{B}_N\ddot{\mathbf{Y}} + \mathbf{Y} = 0. \tag{10}$$

The representation of the waveform in the form $y_k = u_k \sin(\omega t + \varphi_0)$, where ω is the natural frequency of the oscillations, gives the relation $\ddot{\mathbf{Y}} = -\omega^2\mathbf{Y}$. Hence from (3) it follows $\mathbf{B}_N\mathbf{Y} = \lambda\mathbf{Y}$, where the oscillation frequency is expressed in terms of the eigenvalues of the matrix $\mathbf{B}_N$: $\lambda = 1/(m\omega^2)$. The problem is reduced to the eigenvalues problem of the flexibility matrix. The eigenvalues of a matrix in the Maple system can be obtained using the special operator *Eigenvalues* from the linear algebra package *LinearAlgebra*. The oscillation frequencies correspond to the eigenvalues: $\omega = \sqrt{1/(m\lambda)}$. The lowest frequency is the first frequency, the lower value of which must be found.

The graph in Figure 2 is plotted at $EF = 1.0 \cdot 10^3 kH$, $a = 3$m.

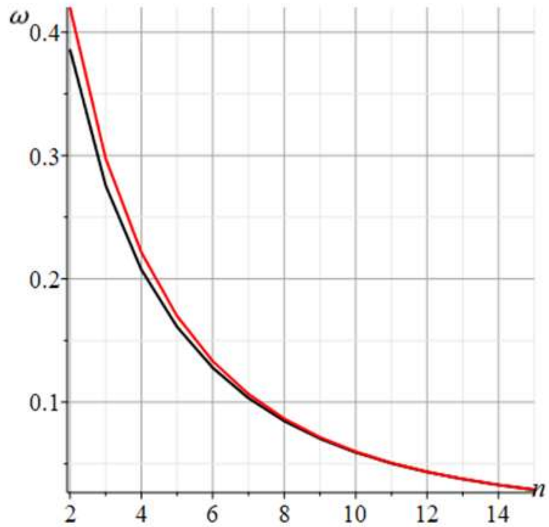

Fig. 2. Dependence of the oscillation frequency (c^{-1}) on the number of panels *n* at *a*=3 m

The numerical value ω_{Nm} of the natural frequency of the system with $N = 3n+1$ degrees of freedom, found as the minimum frequency of the full frequency spectrum, almost completely coincides with the Dunkerley approximation, the curves completely merge. For a refined estimate of the degree of the obtained approximation, we introduce the value of the relative error $\varepsilon = (\omega_{Nm} - \omega_D) / \omega_{Nm}$.

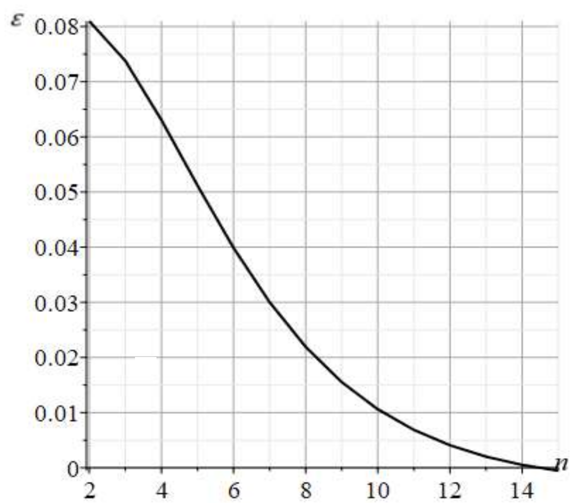

Fig. 3. The error of the Donkerley approximation as a function of the number of panels at *h*=4m, *a*=3m.

Calculations using formula (7) show that the oscillation frequency depends non-linearly on the height of the panel *h* (Fig. 4). The graphs are constructed according to the analytical solution with the same values of the masses and stiffness of the rods as the previous graphs. The curves detect maxima, which allows us to use the obtained solution in optimization problems for choosing the lower frequency of natural oscillations. As the number of panels increases, the maximum shifts towards higher truss heights.

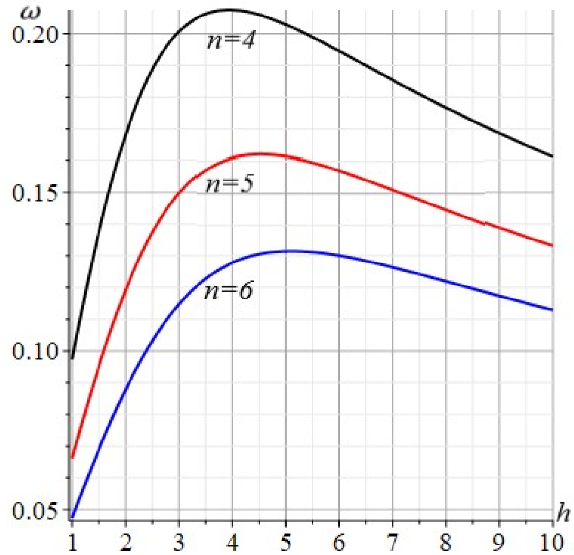

Fig. 4. Oscillation frequency (c^{-1}) as a function of height h (m)

Conclusion

For a planar cantilever truss, an estimate of the lowest natural oscillation frequency is obtained. The accuracy of the solution from the comparison with the numerical one turned out to be quite acceptable. As the number of panels increases, the error tends to zero. The solution has the form of a simple formula and can be used to quickly evaluate the frequency characteristics of the structure. A separate result of the work is the given algorithm for allocating the frequency estimate, which allows its application in other similar problems for statically definable rod systems.

This research has been supported by the Interdisciplinary Scientific and Educational School of Moscow University «Fundamentaland Applied Space Research».

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ЗАВИСИМОСТЬ СОБСТВЕННОЙ ЧАСТОТЫ КОЛЕБАНИЙ ПОЛУРАСКОСНОЙ КОНСОЛИ ОТ ЧИСЛА ПАНЕЛЕЙ

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Рассмотрена плоская консольная ферма, масса которой распределена по узлам. Решение задачи с помощью метода индукции находится в системе компьютерной математики Maple. Для определения усилий в стержнях используется метод вырезания узлов. Основная частота колебаний определяется по методу Донкерлея. Погрешность полученной зависимости сравнивается с численным решением задачи о колебании системы грузов и оказывается весьма небольшой, уменьшаясь до 1% с увеличением числа панелей.

Ключевые слова: консоль, собственная частота колебаний, Maple, индукция, число панелей