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Анализ собственных частот колебаний плоской фермы с произвольным числом панелей

М.Н. Кирсанов, Д. В. Тиньков

Национальный исследовательский университет "Московский энергетический институт" (НИУ "МЭИ"), 111250, г. Москва, Красноказарменная ул., д. 14

АННОТАЦИЯ

Введение: Аналитические решения задач строительной механики не только альтернативный подход к решению проблем прочности, надежности и динамики сооружений, но и возможность для простых оценок работоспособности и оптимизации конструкций. Частотный анализ плоских ферм, наиболее часто применяющихся в строительстве и машиностроении, является важной составной частью исследования сооружений.

Цели — разработка алгоритма трехпараметрической индукции для вывода аналитической зависимости собственных частот колебаний фермы от числа панелей.

Материалы и методы: Рассмотрена плоская статически определимая ферма с одной дополнительной внешней связью и сдвоенными раскосами. Инерционные свойства фермы моделируются точечными массами, расположенными в узлах нижнего прямолинейного пояса фермы. У каждой массы предполагается наличие только одной вертикальной степени свободы. Жесткость всех стержней фермы принимается одинаковой. Ставится задача получения аналитических зависимостей частот колебаний предложенной модели фермы от числа панелей. Вывод искомых формул производится методом индукции в три этапа - по номерам строк и столбцов матрицы податливости, вычисленной по формуле Максвелла-Мора и по числу панелей. Для нахождения общих членов полученных последовательностей коэффициентов применялся аппарат составления и решения рекуррентных уравнений системы компьютерной математики Maple. Задача определения частот свелась к задаче на собственные значения бисимметричной матрицы.

Результаты: Для элементов матрицы податливости найдены общие формулы, по котором составлены и решены частотные уравнения. Показано, что в спектрах частот ферм с различным числом панелей всегда присутствует одна общая частота (средняя частота), располагающаяся в середине спектра. Найдено выражение для максимального значения средней частоты колебаний как функции высоты фермы.

Выводы: Предложенная схема фермы несмотря на свою внешнюю статическую неопределимость и решетку, не позволяющую применять для расчета усилий такие методы как метод вырезания узлов и метод сечений, допускает аналитические решения для частот собственных колебаний грузов в узлах. Полученные формулы имеют достаточно простой вид, а

некоторые общие свойства, такие как совпадения частот для разных чисел панелей и наличие аналитически рассчитываемого максимума функции средней частоты от высоты фермы, делают это решение удобным для практических оценок конструкций.

Ключевые слова: частота колебаний, ферма, индукция, Maple, аналитическое решение

Analysis of the natural frequencies of oscillations of a planar truss with an arbitrary number of panels

Mikhail N. Kirsanov, Dmitriy V. Tinkov

National Research University "Moscow Power Engineering Institute" (MPEI), 14 Krasnokazarmennaya st., Moscow, 111250, Russian Federation

ABSTRACT

Introduction. Analytical solutions for problems of structural mechanics are not only an alternative approach to solving problems of strength, reliability and dynamics of structures, but also the possibility for simple performance evaluations and optimization of structures. Frequency analysis of planar trusses, most often used in construction and engineering, is an important part of the study of structures.

Objectives — development of a three-parameter induction algorithm for deriving the analytical dependence of the natural oscillation frequencies of the truss on the number of panels.

Materials and methods. A flat, statically definable truss with one additional external link and double braces has been considered. The inertia properties of the truss are modeled by point masses located in the nodes of the lower straight truss belt. Each mass is assumed to have only one vertical degree of freedom. The stiffness of all truss rods is assumed to be the same. The task is to obtain analytical dependences of the oscillation frequencies of the proposed truss model on the number of panels. The derivation of the desired formulas is performed by the method of induction in three stages - according to the numbers of rows and columns of the compliance matrix, calculated using the Maxwell-Mohr formula and the number of panels. To find common members of the obtained sequences of coefficients, an apparatus was used to compile and solve the recurrent equations of the Maple computer mathematics system. The task of determining frequencies has been reduced to the eigenvalue problem of a bisymmetric matrix.

Results. For the elements of the compliance matrix, general formulas have been found, according to which the frequency equations are compiled and solved. It is shown that in the frequency spectra of trusses with different numbers of panels there is always one common frequency (middle frequency) located in the middle of the spectrum. An expression is found for the maximum value of the average oscillation frequency as a function of the height of the truss.

Conclusions. The proposed truss scheme, despite its external static indeterminacy and the lattice, which does not allow for the calculation of forces by such methods as the method of cutting nodes and the cross section method, allows analytical solutions for the natural frequencies of loads in the nodes. The obtained formulas have a rather simple form, and some general properties, such as frequency coincidences for different numbers of panels and the presence of an analytically calculated maximum of the average frequency function of the truss height, make this solution convenient for practical structural evaluations.

KEYWORDS: oscillation frequency, truss, induction, Maple, analytical solution

INTRODUCTION

Modern computer systems of symbolic mathematics make it possible to find analytical solutions to problems of structural mechanics as an alternative approach to solving the problems of strength, reliability, and dynamics of structures [1–7]. In [8–13], the induction method involving the operators of the Maple system obtained formulas for the dependence of the deflection of planar trusses on the number of panels. Analytical solutions of problems on the oscillation of a load with one degree on a truss with an arbitrary number of panels were obtained in [14–17]. A more accurate picture of the dynamics of trusses can be given by analyzing a truss model with a distributed mass, or at least with a mass distributed over the nodes of the lower belt. The main difficulty in obtaining such solutions is to determine the rigidity of the structure. In the elastic stage of the truss rods with small oscillations to find the compliance matrix inverse to the stiffness matrix, a very convenient method is to use the Maxwell – Mohr formula. The forces in the truss rods included in this formula in solving the problems of the stiffness of the arches [18-23], lattice [24-30] and spatial trusses [31-33] were determined on the basis of the program [8-11] written in Maple language on basis of the cutting knots method. The main limitation for the analytical method, designed for the analysis of systems with an arbitrary number of panels, is the regularity of the truss schemes [34,35]. If there are periodically repeating structures in the structure, for example, panels, then the induction method is applicable to such trusses. *Hutchinson R. G.* and *Fleck N.A.* [36,37] dealt with the problems of the existence of regular statically definable schemes, and methods of their calculation. Some particular problems of periodic trusses are considered in [38].

MATERIALS AND METHODS

Consider a truss with double braces and an additional horizontal external link on the left support (Fig. 1). The truss has $2n$ panels and $n_s = 16n + 4$ rods, including four rods, modeling the supports. It is assumed that all rods have the same stiffness *EF*. An analytical solution of the problem of the deflection of this truss for an arbitrary number of panels is given in [39]. Solutions for the case of uniform load over the nodes of the upper and lower belts are obtained by generalizing a number of solutions for trusses with the number *n* of panels in half span from 1 to 10:

$$\Delta = PC_n\left(a^3 + 2bh^2 + c^3\right)/\left(h^2 EF\right),$$

where $c = \sqrt{a^2 + h^2}$ is the length of the brace, C_n is a coefficient depending on the type of load. The Maxwell – Mohr formula was used to calculate the deflection.

$$\Delta = P\sum_{j=1}^{n_s-4} \frac{S_j s_j l_j}{EF},$$

where l_j and S_j is the length and force in the *j* th rod from the action of the load, s_j is the force from a single vertical force applied to the central node in the lower belt. The forces in the rods were determined by cutting the nodes from the system of

linear equilibrium equations compiled for all the nodes of the structure as a whole, which made it possible to overcome the external static indeterminacy of the truss. The solution of the system of linear equations was in symbolic form according to a program written in the language of computer mathematics Maple.

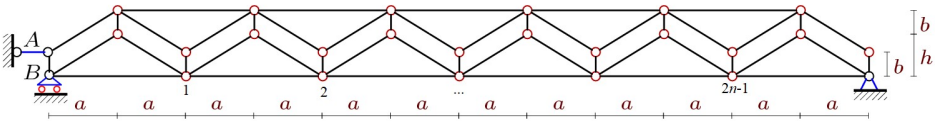

Fig. 1. The truss scheme, $n = 3$

To derive a formula for the dependence of the frequency of oscillations of loads located in the nodes of the lower belt on the number of panels and the geometry of the structure, we will use the same method. The equations of vertical oscillations of cargo we write in the form

$$[M_n]\ddot{\vec{Y}} + [D_n]\vec{Y} = 0, \qquad (1)$$

where $[M_n]$ is the matrix of inertia, $\vec{Y}$ is the vector of vertical displacements of masses, $[D_n]$ is the stiffness matrix, $\ddot{\vec{Y}}$ is the vector of accelerations. If the masses of the loads are the same, then the inertia matrix is diagonal:

$$M_n = \begin{bmatrix} m & 0 & \dots & 0 \\ 0 & m & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & m \end{bmatrix}.$$

The compliance matrix $[B_n]$, the inverse stiffness matrix $[D_n]$, has the following elements:

$$b_{i,j} = \sum_{k=1}^{n_s-4} S_k^{(i)} S_k^{(j)} l_k / (EF), \qquad (2)$$

where $S_k^{(i)}$ is the force in the rod k from the action of a single vertical force at node i, l_k is the length of the rod. Multiplying (1) from the left by the matrix $[B_n]$, we get the equation $m[B_n]\ddot{\vec{Y}} + \vec{Y} = 0$. The vector of vertical displacements will be represented as a periodic function $\vec{Y} = \vec{A}\sin(\omega t + \varphi_0)$. From here, taking into account the relation $\ddot{\vec{Y}} = -\omega^2 \vec{Y}$, we obtain an eigenvalue problem $[B_n]\vec{Y} = \lambda \vec{Y}$, where

$$\lambda = 1/(m\omega^2). \qquad (3)$$

Thus, to solve the problem, it is necessary to obtain analytical expressions for the matrix members $[B_n]$. This matrix is symmetric not only with respect to the main diagonal (due to symmetry (2) with respect to i and j), but also with respect to the secondary diagonal. The last property is related to the symmetry of the structure. The vertical displacement of the node k from the action of a unit load at the node $2n - k$ is equal to the displacement of the node $2n - k$ from the unit force at the node k.

Bisymmetric matrices were studied in [40]. When n = 3 we have the following form of the matrix

$$[B_3]=\frac{\eta}{n}\begin{bmatrix}205 & 308 & 315 & 250 & 137\\ * & 520 & 558 & 452 & *\\ * & * & 657 & * & *\\ * & * & * & * & *\\ * & * & * & * & *\end{bmatrix},$$

where denoted $\eta=\left(a^3+2bh^2+c^3\right)/\left(3h^2EF\right)$, and the * symbol denotes elements whose values follow from the properties of the matrix symmetry. This kind of result allows, in the decision process, to calculate by the formula not for all values of i,j = 1,2,...,$2n$ – 1, but only for j = 1, ..., n, $i = j$ ... $2n - j$, which significantly reduces conversion time. To obtain the common members of the sequences in the rows of the matrix $[B_n]$, we use the **rff_findrecur** operator of the Maple system's **genfunc** package, which returns a recurrent equation that is satisfied by the sequence members. Then the **rsolve** operator gives a solution to the equation defining the common term of the sequence. The result can be obtained if the sequence under investigation has a sufficient length. This task requires a sequence of at least eight. Therefore, all calculations must be started from the trusses, the number of panels is more than four. For the first row (j = 1) of the matrix with n = 5, consisting of elements 657, 1128, 1407, 1518, 1485, 1332, 1083, 762, 393, we have the equation

$$b_{1,i}=4b_{1,i-1}-6b_{1,i-2}+4b_{1,i-3}-b_{1,i-4},\ i=1,...,2n-1.$$

The solution of this equation has the form $b_{1,i}=4i^3-120i^2+803i-30.$ Similarly for other lines

$$b_{2,i+1}=4i^3-108i^2+587i+549,\ i=1,...,2n-3,$$
$$b_{3,i+2}=4i^3-96i^2+403i+872,i=1,...,2n-5,$$
$$b_{4,i+3}=4i^3-84i^2+251i+987,\ i=1,...,2n-7.$$

In the general case, for arbitrary j, we have an expression $b_{j,i+j-1}=4i^3-\alpha_{2,5}i^2+\alpha_{1,5}i-\alpha_{0,5}$, where the coefficients $\alpha_{2,5}$, $\alpha_{1,5}$, $\alpha_{0,5}$ are to be determined. The sequence of coefficients with i^2 has a fairly obvious common term $\alpha_{2,5}=12j-132$. For other sequences, the **rgf_findrecur** and **rsolve** operators are required:

$$\alpha_{1,5}=16j^2-264j+1051,$$
$$\alpha_{0,5}=8j^3-176j^2+1051j-913.$$

Solutions are obtained for n = 5. To generalize the solution to an arbitrary number of panels, it is required to repeat the output for other values of n. Omitting the intermediate results we give the corresponding expressions:

$$n = 6:$$
$$\alpha_{2,5} = 12j - 156,$$
$$\alpha_{1,5} = 16j^2 - 312j + 1451,$$
$$\alpha_{0,5} = 8j^3 - 208j^2 + 1451j - 1287,$$

$$n = 7:$$
$$\alpha_{2,5} = 12j - 180,$$
$$\alpha_{1,5} = 16j^2 - 360j + 1915,$$
$$\alpha_{0,5} = 8j^3 - 240j^2 + 1915j - 1725,$$
$$...$$

Summarizing these expressions for the general case, we obtain

$$\alpha_{2,n} = 12(j - 1 - 2n),$$
$$\alpha_{1,n} = 16j^2 - 24(1 + 2n)j + 32n^2 + 48n + 11,$$
$$\alpha_{0,n} = 8j^3 - 16(1 + 2n)j^2 + (32n^2 + 48n + 11)j - 32n^2 - 22n - 3.$$

Together with the expression

$$b_{j,i+j-1} = 4i^3 - \alpha_{2,n}i^2 + \alpha_{1,n}i - \alpha_{0,n}$$

these coefficients constitute the main basic part of the matrix, the reflection of which relative to the main and secondary diagonal gives the full matrix, whose eigenvalues give the solution. For reflection on the main diagonal, use the ratios

$$b_{i,j} = b_{i,j},\ j = 1,...,2n-1,\ i = j+1,...,2n-1.$$

Elements that are symmetrical with respect to the secondary diagonal are obtained using the relations

$$b_{i,2n-j} = b_{2n-i,j},\ j = 1,...,2n-1,\ i = j+1,...,2n-1.$$

Results

The result of induction on the three parameters were the expressions for the elements of the matrix, the eigenvalues of which give the oscillation frequencies of the truss, whose inertial properties are modeled by weights in the nodes of the lower belt, which allow only vertical displacements. The oscillation frequencies are determined by the formula (3) as applied to trusses with given elastic and geometric characteristics. For $n = 2$, we have the matrix

$$[B_2] = \frac{\eta}{n}\begin{bmatrix} 75 & 90 & 57 \\ 90 & 132 & 90 \\ 57 & 90 & 75 \end{bmatrix}$$

The eigenvalues of the matrix are

$$\lambda_1 = 9\eta,\ \lambda_{2,3} = 3(22 \pm 15\sqrt{2})\eta. \tag{4}$$

Compliance matrix at n=3:

$$[B_3] = \frac{\eta}{n}\begin{bmatrix} 205 & 308 & 315 & 250 & 137 \\ 308 & 520 & 558 & 452 & 250 \\ 315 & 558 & 657 & 558 & 315 \\ 250 & 452 & 558 & 520 & 308 \\ 137 & 250 & 315 & 308 & 205 \end{bmatrix},$$

Eigenvalues of this matrix:

$$\lambda_1 = 9\eta,\ \lambda_2 = 42\eta,\ \lambda_3 = 10\eta/3,\ \lambda_{2,3} = 6(54 \pm 31\sqrt{3})\eta. \tag{5}$$

When n = 4, the set of seven eigenvalues consists of three values (4) and

$$\lambda_{4,5} = 3\left(172 \pm 118\sqrt{2} \pm \sqrt{57236 \pm 4062\sqrt{2}}\ \right)\eta,$$

$$\lambda_{6,7} = 3\left(172 \pm 118\sqrt{2} \mp \sqrt{57236 \pm 4062\sqrt{2}}\ \right)\eta.$$

It is noted that for all numbers of panels n in the spectrum of natural frequencies there is a value $\lambda_1 = 9\eta$, and for even n the values $\lambda_{2,3} = 3(22 \pm 15\sqrt{2})\eta$ are also included in the spectrum. In addition, calculations show that for numbers n multiple of three, the spectrum includes values (5), and for numbers n multiple of four, the spectrum includes values of the spectrum for n = 4. It can be assumed that a more general statement is true: the frequency spectrum of a truss with the number $n = k_1 k_2$ of panels includes formulas for the frequency spectra of trusses with the number of panels k_1 and k_2. The assertion is verified for a number of numbers, but in the general case it still requires proof. For n = 5, the curves of frequency versus truss height reveal a maximum (Fig. 2)

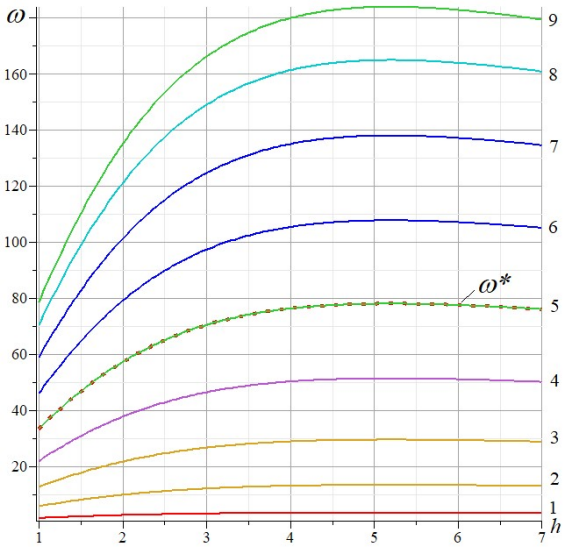

Fig. 2. The natural frequencies of the truss (rad /s), depending on the height h with $n = 5$

This solution was obtained for mass m = 100 kg, stiffness $EF = 2{,}0 \cdot 10^7$ N, panel length a = 3 m and height of struts b = 1 m. The following regularity is noted in the graphs: the frequency obtained from the eigenvalue $\lambda_1 = 9\eta$ present in the solutions for any n is located in the middle of the spectrum . This is confirmed by graphs plotted for other values of n. Analytical representation of the solution allows finding the exact values of the extremal point. From the condition $d\omega^*/dh = 0$ where

$$\omega^* = 1/\sqrt{m\lambda_1} = \frac{h\sqrt{3EF}}{3\sqrt{m(a^3 + 2bh^2 + c^3)}}$$

that the maximum frequency $\omega^*_{\max} = \dfrac{\sqrt{3EF}}{3\sqrt{m(3a + 2b)}}$ is reached when the height value $h = \sqrt{3}\,a$.

CONCLUSIONS

Methods of symbolic mathematics made it possible to find not only exact expressions for the elements of the matrix that defines the eigenfrequencies of free oscillations of loads in the truss nodes, but also to obtain analytical expressions for the frequencies. In a numerical analysis of the results obtained, it was also found that, regardless of the number of panels, the design under consideration has the same oscillation frequency located in the middle of the spectrum. The comparative

simplicity of the solution also allowed us to find the exact expression for the extreme point on the graph of the dependence of the average frequency on the height of the truss. A significant simplification in the derivation of the desired formulas turned out to be a technique based on the bisymmetric properties of the compliance matrix, which reduces the calculation of all elements of the matrix to the calculation of only the elements of its basic triangle with the subsequent reflection of elements relative to the main and secondary diagonal. Certainly, the experience of the authors in solving the problems of deflection of statically definable flat trusses in analytical form by the method of induction [8–11] was useful for successful work. Compared to these tasks, the solved problem of the oscillation frequencies of a system with many degrees of freedom is significantly more difficult due to the three levels of induction in rows and columns of the matrix and in the number of panels. So if in the simple problem of deflection of the generalization of the result to an arbitrary number of panels, it is necessary in analytical form to solve on average k problems about the forces in the rods and the deflection of the truss, then with triple induction of such problems already k^3.

Verification of the results obtained numerically.

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О б а в т о р а х : **Кирсанов Михаил Николаевич** — доктор физико-математических наук, профессор кафедры робототехники, мехатроники, динамики и прочности машин, **Национальный исследовательский университет «Московский энергетический институт» (НИУ «МЭИ»),** 111250, г. Москва, ул. Красноказарменная, д. 14, c216@ya.ru.

Тиньков Дмитрий Владимирович — аспирант кафедры робототехники, мехатроники, динамики и прочности машин, **Национальный исследовательский университет «Московский энергетический институт» (НИУ «МЭИ»),** 111250, г. Москва, ул. Красноказарменная, д. 14, c216@ya.ru.