

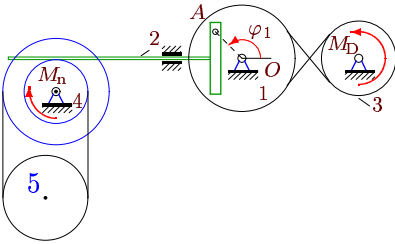
Кулиса. Уравнение Лагранжа

Кулисный механизм расположен в вертикальной плоскости и состоит из однородных цилиндров, блоков (радиус инерции i_k), штока и груза¹. Используя уравнение Лагранжа 2-го рода, получить уравнение движения механизма. Найти значение углового ускорения $\ddot{\varphi}_1$ при $t = 0$. Кинетическую энергию представить в форме $T = (\dot{\varphi}^2/2)(A + B \sin^2 \varphi)$ или $T = (\dot{\varphi}^2/2)(A + B \cos^2 \varphi)$ (варианты помечены *)

Кирсанов М.Н. **Решebник. Теоретическая механика**/Под ред. А. И. Кириллова.– М.: ФИЗМАТЛИТ, 2008. — 384 с. (с.307.)

Задача D-31.1.

Абалин Максим



$$M_{D_z} = M_0 - k\omega_{3z}, \quad M_{n_z} = -\mu\omega_{4z},$$

$$M_0 = 9 \text{ Нм}, \quad k = 11 \text{ Нмс},$$

$$\varphi_{1,0} = 1.1, \quad \omega_{1z,0} = 0.1 \frac{1}{\text{с}},$$

$$\mu = 14 \text{ Нмс}, \quad I_1 = 6 \text{ кгм}^2,$$

$$m_2 = 14 \text{ кг}, \quad m_3 = 32 \text{ кг}, \quad m_4 = 24 \text{ кг},$$

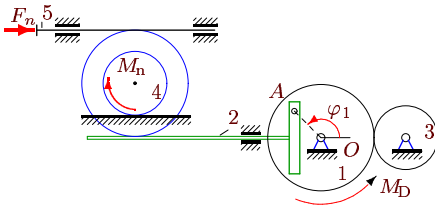
$$m_5 = 30 \text{ кг}, \quad R_1 = 34 \text{ см}, \quad r_1 = 23 \text{ см},$$

$$R_3 = 24 \text{ см}, \quad R_4 = 20 \text{ см}, \quad r_4 = 12 \text{ см}, \quad i_4 = 13 \text{ см},$$

$$r_5 = 16 \text{ см}.$$

Задача D-31.2.

Абдулвалиев Роман



$$M_{D_z} = M_0 - k\omega_{1z}, \quad M_{n_z} = -\mu\omega_{4z},$$

$$F_{n_x} = -\nu v_{5x}, \quad \varphi_{1,0} = 1.3, \quad \omega_{1z,0} = 0.1 \frac{1}{\text{с}},$$

$$M_0 = 12 \text{ Нм}, \quad k = 11 \text{ Нмс},$$

$$\nu = 30 \text{ Нс/м}, \quad \mu = 13 \text{ Нмс},$$

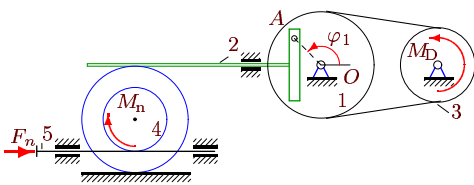
$$I_1 = 15 \text{ кгм}^2, \quad m_2 = 16 \text{ кг}, \quad m_3 = 34 \text{ кг},$$

$$m_4 = 26 \text{ кг}, \quad R_1 = 34 \text{ см}, \quad r_1 = 23 \text{ см},$$

$$R_3 = 24 \text{ см}, \quad R_4 = 20 \text{ см}, \quad r_4 = 12 \text{ см}, \quad i_4 = 15 \text{ см}.$$

Задача D-31.3.

Антонова Вера



$$M_{D_z} = M_0 - k\omega_{3z}, \quad M_{n_z} = -\mu\omega_{4z},$$

$$F_{n_x} = -\nu v_{5x}, \quad \varphi_{1,0} = 1.3, \quad \omega_{1z,0} = 0.2 \frac{1}{\text{с}},$$

$$M_0 = 10 \text{ Нм}, \quad k = 12 \text{ Нмс},$$

$$\nu = 8 \text{ Нс/м}, \quad \mu = 12 \text{ Нмс},$$

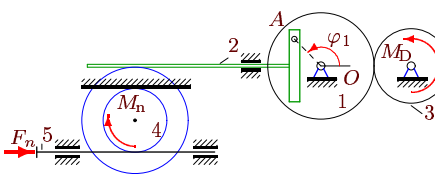
$$I_1 = 9 \text{ кгм}^2, \quad m_2 = 16 \text{ кг}, \quad m_3 = 34 \text{ кг},$$

$$m_4 = 26 \text{ кг}, \quad R_1 = 35 \text{ см}, \quad r_1 = 24 \text{ см},$$

$$R_3 = 25 \text{ см}, \quad R_4 = 20 \text{ см}, \quad r_4 = 12 \text{ см}, \quad i_4 = 15 \text{ см}.$$

Задача D-31.4.

Большаков Павел



$$M_{D_z} = M_0 - k\omega_{3z}, \quad M_{n_z} = -\mu\omega_{4z},$$

$$F_{n_x} = -\nu v_{5x}, \quad \varphi_{1,0} = 1.5, \quad \omega_{1z,0} = 0.1 \frac{1}{\text{с}},$$

$$M_0 = 11 \text{ Нм}, \quad k = 11 \text{ Нмс},$$

$$\nu = 35 \text{ Нс/м}, \quad \mu = 10 \text{ Нмс},$$

$$I_1 = 8 \text{ кгм}^2, \quad m_2 = 18 \text{ кг}, \quad m_3 = 36 \text{ кг},$$

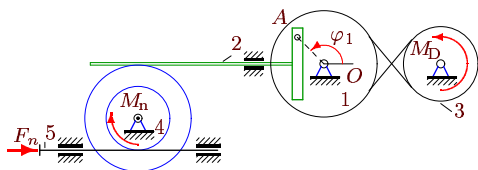
$$m_4 = 28 \text{ кг}, \quad R_1 = 34 \text{ см}, \quad r_1 = 23 \text{ см},$$

$$R_3 = 24 \text{ см}, \quad R_4 = 20 \text{ см}, \quad r_4 = 12 \text{ см}, \quad i_4 = 17 \text{ см}.$$

¹В некоторых вариантах содержатся не все элементы.

Задача D-31.5.

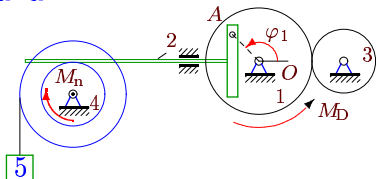
Воропай Руслан



$$\begin{aligned} M_{Dz} &= M_0 - k\omega_{3z}, M_{nz} = -\mu\omega_{4z}, \\ F_{nx} &= -\nu v_{5x}, \varphi_{1,0} = 1.1, \omega_{1z,0} = 0.2 \frac{1}{c}, \\ M_0 &= 9 \text{ Нм}, k = 12 \text{ Нмс}, \\ \nu &= 8 \text{ кНс/м}, \mu = 14 \text{ Нмс}, \\ I_1 &= 6 \text{ кгм}^2, m_2 = 14 \text{ кг}, m_3 = 32 \text{ кг}, \\ m_4 &= 24 \text{ кг}, R_1 = 35 \text{ см}, r_1 = 24 \text{ см}, \\ R_3 &= 25 \text{ см}, R_4 = 20 \text{ см}, r_4 = 12 \text{ см}, i_4 = 13 \text{ см}. \end{aligned}$$

Задача D-31.6.

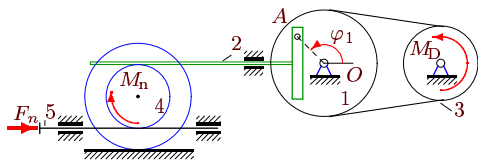
Ершов Леонид



$$\begin{aligned} M_{Dz} &= M_0 - k\omega_{1z}, M_{nz} = -\mu\omega_{4z}, \\ M_0 &= 11 \text{ Нм}, k = 12 \text{ Нмс}, \\ \varphi_{1,0} &= 1.2, \omega_{1z,0} = 0.2 \frac{1}{c}, \\ \mu &= 13 \text{ Нмс}, I_1 = 11 \text{ кгм}^2, \\ m_2 &= 15 \text{ кг}, m_3 = 33 \text{ кг}, m_4 = 25 \text{ кг}, \\ m_5 &= 5 \text{ кг}, R_1 = 35 \text{ см}, r_1 = 24 \text{ см}, \\ R_3 &= 25 \text{ см}, R_4 = 20 \text{ см}, r_4 = 12 \text{ см}, i_4 = 14 \text{ см}. \end{aligned}$$

Задача D-31.7.

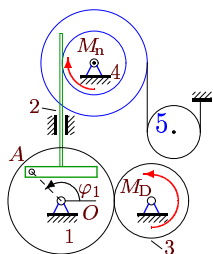
Казбаев Владимир



$$\begin{aligned} M_{Dz} &= M_0 - k\omega_{3z}, M_{nz} = -\mu\omega_{4z}, \\ F_{nx} &= -\nu v_{5x}, \varphi_{1,0} = 1.2, \omega_{1z,0} = 0.5 \frac{1}{c}, \\ M_0 &= 9 \text{ Нм}, k = 15 \text{ Нмс}, \\ \nu &= 8 \text{ кНс/м}, \mu = 13 \text{ Нмс}, \\ I_1 &= 7 \text{ кгм}^2, m_2 = 15 \text{ кг}, m_3 = 33 \text{ кг}, \\ m_4 &= 25 \text{ кг}, R_1 = 38 \text{ см}, r_1 = 27 \text{ см}, \\ R_3 &= 28 \text{ см}, R_4 = 20 \text{ см}, r_4 = 12 \text{ см}, i_4 = 14 \text{ см}. \end{aligned}$$

Задача D-31.8.

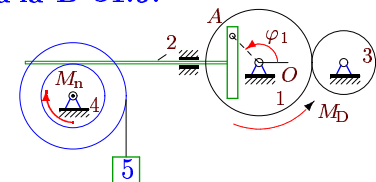
Кириенко Дмитрий



$$\begin{aligned} M_{Dz} &= M_0 - k\omega_{3z}, M_{nz} = -\mu\omega_{4z}, \\ M_0 &= 11 \text{ Нм}, k = 12 \text{ Нмс}, \\ \varphi_{1,0} &= 1.5, \omega_{1z,0} = 0.2 \frac{1}{c}, \\ \mu &= 10 \text{ Нмс}, I_1 = 8 \text{ кгм}^2, \\ m_2 &= 18 \text{ кг}, m_3 = 36 \text{ кг}, m_4 = 28 \text{ кг}, \\ m_5 &= 5 \text{ кг}, R_1 = 35 \text{ см}, r_1 = 24 \text{ см}, \\ R_3 &= 25 \text{ см}, R_4 = 20 \text{ см}, r_4 = 12 \text{ см}, i_4 = 17 \text{ см}, \\ r_5 &= 10 \text{ см}. \end{aligned}$$

Задача D-31.9.

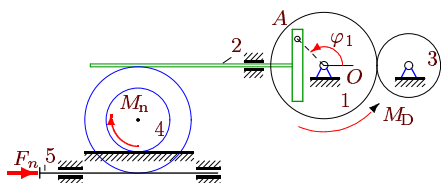
Китайкина Ирина



$$\begin{aligned} M_{Dz} &= M_0 - k\omega_{1z}, M_{nz} = -\mu\omega_{4z}, \\ M_0 &= 12 \text{ Нм}, k = 15 \text{ Нмс}, \\ \varphi_{1,0} &= 1.3, \omega_{1z,0} = 0.5 \frac{1}{c}, \\ \mu &= 12 \text{ Нмс}, I_1 = 15 \text{ кгм}^2, \\ m_2 &= 16 \text{ кг}, m_3 = 34 \text{ кг}, m_4 = 26 \text{ кг}, \\ m_5 &= 6 \text{ кг}, R_1 = 38 \text{ см}, r_1 = 27 \text{ см}, \\ R_3 &= 28 \text{ см}, R_4 = 20 \text{ см}, r_4 = 12 \text{ см}, i_4 = 15 \text{ см}. \end{aligned}$$

Задача D-31.10.

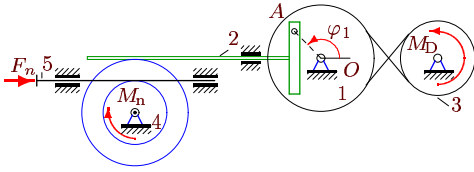
Косарева Елена



$$\begin{aligned} M_{Dz} &= M_0 - k\omega_{1z}, M_{nz} = -\mu\omega_{4z}, \\ F_{nx} &= -\nu v_{5x}, \varphi_{1,0} = 1.4, \omega_{1z,0} = 0.3 \frac{1}{c}, \\ M_0 &= 13 \text{ Нм}, k = 13 \text{ Нмс}, \\ \nu &= 8 \text{ кНс/м}, \mu = 11 \text{ Нмс}, \\ I_1 &= 19 \text{ кгм}^2, m_2 = 17 \text{ кг}, m_3 = 35 \text{ кг}, \\ m_4 &= 27 \text{ кг}, R_1 = 36 \text{ см}, r_1 = 25 \text{ см}, \\ R_3 &= 26 \text{ см}, R_4 = 20 \text{ см}, r_4 = 12 \text{ см}, i_4 = 16 \text{ см}. \end{aligned}$$

Задача D-31.11.

Костин Андрей



$$M_{Dz} = M_0 - k\omega_{3z}, M_{nz} = -\mu\omega_{4z},$$

$$F_{nx} = -\nu v_{5x}, \varphi_{1,0} = 1.1, \omega_{1z,0} = 0.3 \frac{1}{c},$$

$$M_0 = 9 \text{ НМ}, k = 13 \text{ НМС},$$

$$\nu = 40 \text{ Нс/М}, \mu = 15 \text{ НМС},$$

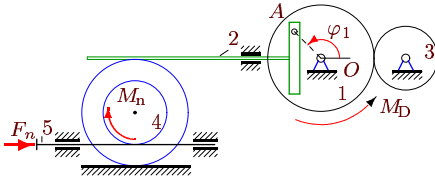
$$I_1 = 6 \text{ кгМ}^2, m_2 = 14 \text{ кг}, m_3 = 32 \text{ кг},$$

$$m_4 = 24 \text{ кг}, R_1 = 36 \text{ см}, r_1 = 25 \text{ см},$$

$$R_3 = 26 \text{ см}, R_4 = 20 \text{ см}, r_4 = 12 \text{ см}, i_4 = 13 \text{ см}.$$

Задача D-31.12.

Любчик Владислав



$$M_{Dz} = M_0 - k\omega_{1z}, M_{nz} = -\mu\omega_{4z},$$

$$F_{nx} = -\nu v_{5x}, \varphi_{1,0} = 1.5, \omega_{1z,0} = 0.3 \frac{1}{c},$$

$$M_0 = 14 \text{ НМ}, k = 13 \text{ НМС},$$

$$\nu = 8 \text{ Нс/М}, \mu = 10 \text{ НМС},$$

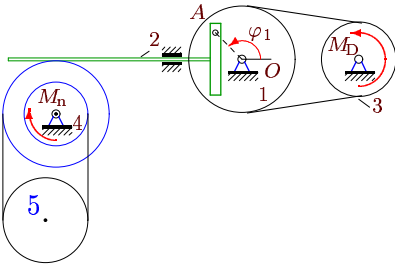
$$I_1 = 23 \text{ кгМ}^2, m_2 = 18 \text{ кг}, m_3 = 36 \text{ кг},$$

$$m_4 = 28 \text{ кг}, R_1 = 36 \text{ см}, r_1 = 25 \text{ см},$$

$$R_3 = 26 \text{ см}, R_4 = 20 \text{ см}, r_4 = 12 \text{ см}, i_4 = 17 \text{ см}.$$

Задача D-31.13.

Маслов Владислав



$$M_{Dz} = M_0 - k\omega_{3z}, M_{nz} = -\mu\omega_{4z},$$

$$M_0 = 9 \text{ НМ}, k = 12 \text{ НМС},$$

$$\varphi_{1,0} = 1.2, \omega_{1z,0} = 0.2 \frac{1}{c},$$

$$\mu = 14 \text{ НМС}, I_1 = 7 \text{ кгМ}^2,$$

$$m_2 = 15 \text{ кг}, m_3 = 33 \text{ кг}, m_4 = 25 \text{ кг},$$

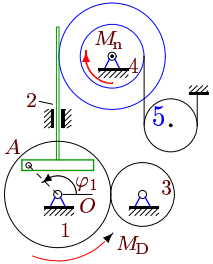
$$m_5 = 40 \text{ кг}, R_1 = 35 \text{ см}, r_1 = 24 \text{ см},$$

$$R_3 = 25 \text{ см}, R_4 = 20 \text{ см}, r_4 = 12 \text{ см}, i_4 = 14 \text{ см},$$

$$r_5 = 16 \text{ см}.$$

Задача D-31.14.

Матвеев Александр



$$M_{Dz} = M_0 - k\omega_{1z}, M_{nz} = -\mu\omega_{4z},$$

$$M_0 = 10 \text{ НМ}, k = 12 \text{ НМС},$$

$$\varphi_{1,0} = 1.1, \omega_{1z,0} = 0.2 \frac{1}{c},$$

$$\mu = 15 \text{ НМС}, I_1 = 7 \text{ кгМ}^2,$$

$$m_2 = 14 \text{ кг}, m_3 = 32 \text{ кг}, m_4 = 24 \text{ кг},$$

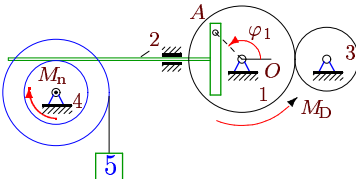
$$m_5 = 5 \text{ кг}, R_1 = 35 \text{ см}, r_1 = 24 \text{ см},$$

$$R_3 = 25 \text{ см}, R_4 = 20 \text{ см}, r_4 = 12 \text{ см}, i_4 = 13 \text{ см},$$

$$r_5 = 13 \text{ см}.$$

Задача D-31.15.

Пиценко Денис



$$M_{Dz} = M_0 - k\omega_{1z}, M_{nz} = -\mu\omega_{4z},$$

$$M_0 = 10 \text{ НМ}, k = 14 \text{ НМС},$$

$$\varphi_{1,0} = 1.1, \omega_{1z,0} = 0.4 \frac{1}{c},$$

$$\mu = 14 \text{ НМС}, I_1 = 7 \text{ кгМ}^2,$$

$$m_2 = 14 \text{ кг}, m_3 = 32 \text{ кг}, m_4 = 24 \text{ кг},$$

$$m_5 = 4 \text{ кг}, R_1 = 37 \text{ см}, r_1 = 26 \text{ см},$$

$$R_3 = 27 \text{ см}, R_4 = 20 \text{ см}, r_4 = 12 \text{ см}, i_4 = 13 \text{ см}.$$

The diagram shows a horizontal bar of length 2, pivoted at its right end. A spring with stiffness k is attached to the left end of the bar. A wheel of radius r is in contact with the bar at a distance 5 from its left end. A horizontal force F_n is applied to the left at this point. The wheel is pivoted at its center O and has a moment M_D applied. The angle between the bar and the horizontal is φ_1 . The wheel is also in contact with a horizontal surface at its bottom point A .

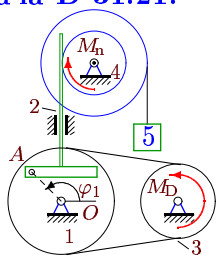
$$\begin{aligned} M_{D_z} &= M_0 - k\omega_{3_z}, M_{n_z} = -\mu\omega_{4_z}, \\ F_{n_x} &= -\nu\nu_{5_x}, \varphi_{1,0} = 1.4, \omega_{1,z,0} = 0.2\frac{1}{c}, \\ M_0 &= 10 \text{ HM}, k = 12 \text{ HMC}, \\ \nu &= 8\text{KHC/M}, \mu = 11 \text{ HMC}, \\ I_1 &= 7 \text{ KGM}^2, m_2 = 17 \text{ KG}, m_3 = 35 \text{ KG}, \\ m_4 &= 27 \text{ KG}, R_1 = 35 \text{ CM}, r_1 = 24 \text{ CM}, \\ R_3 &= 25 \text{ CM}, R_4 = 20 \text{ CM}, r_4 = 12 \text{ CM}, i_4 = 16 \text{ CM}. \end{aligned}$$

The diagram shows a mechanical system with three wheels (1, 2, and 3) and a horizontal bar. Wheel 1 is at the center O, wheel 2 is at point A, and wheel 3 is at point B. A horizontal bar connects the centers of the three wheels. A force F_n is applied to the left of wheel 1, and a moment M_D is applied to wheel 1. The wheels are in contact with horizontal surfaces.

$$\begin{aligned} M_{D_z} &= M_0 - k\omega_{1z}, M_{n_z} = -\mu\omega_{4z}, \\ F_{n_x} &= -\nu\nu_{5x}, \varphi_{1,0} = 1.2, \omega_{1z,0} = 0.2\frac{1}{c}, \\ M_0 &= 11 \text{ HM}, k = 12 \text{ HMC}, \\ \nu &= 25 \text{ HC/M}, \mu = 13 \text{ HMC}, \\ I_1 &= 11 \text{ KGM}^2, m_2 = 15 \text{ KG}, m_3 = 33 \text{ KG}, \\ m_4 &= 25 \text{ KG}, R_1 = 35 \text{ CM}, r_1 = 24 \text{ CM}, \\ R_3 &= 25 \text{ CM}, R_4 = 20 \text{ CM}, r_4 = 12 \text{ CM}, i_4 = 14 \text{ CM}. \end{aligned}$$
$$\begin{aligned} M_{D_z} &= M_0 - k\omega_{3_z}, M_{n_z} = -\mu\omega_{4_z}, \\ \varphi_{1,0} &= 1.5, \omega_{1_z,0} = 0.2\frac{1}{c}, \\ M_0 &= 13 \text{ HM}, k = 12 \text{ HMC}, \\ \mu &= 11 \text{ HMC}, \\ I_1 &= 18 \text{ KGM}^2, m_2 = 18 \text{ KG}, m_3 = 36 \text{ KG}, \\ m_4 &= 28 \text{ KG}, R_1 = 35 \text{ CM}, r_1 = 24 \text{ CM}, \\ R_3 &= 25 \text{ CM}, R_4 = 20 \text{ CM}, r_4 = 12 \text{ CM}, i_4 = 17 \text{ CM}. \end{aligned}$$
$$\begin{aligned} M_{D_z} &= M_0 - k\omega_{3_z}, M_{n_z} = -\mu\omega_{4_z}, \\ F_{n_x} &= -\nu v_{5_x}, \varphi_{1,0} = 1.1, \omega_{1,z,0} = 0.2 \frac{1}{c}, \\ M_0 &= 8 \text{ HM}, k = 12 \text{ HMC}, \\ \nu &= 30 \text{ HC/M}, \mu = 15 \text{ HMC}, \\ I_1 &= 5 \text{ KGM}^2, m_2 = 14 \text{ KG}, m_3 = 32 \text{ KG}, \\ m_4 &= 24 \text{ KG}, R_1 = 35 \text{ CM}, r_1 = 24 \text{ CM}, \\ R_3 &= 25 \text{ CM}, R_4 = 20 \text{ CM}, r_4 = 12 \text{ CM}, i_4 = 13 \text{ CM}. \end{aligned}$$
$$\begin{aligned} M_{D_z} &= M_0 - k\omega_{3_z}, M_{n_z} = -\mu\omega_{4_z}, \\ M_0 &= 12 \text{ H}_{\text{M}}, k = 13 \text{ H}_{\text{MC}}, \\ \varphi_{1,0} &= 1.4, \omega_{1_z,0} = 0.3 \frac{1}{c}, \\ \mu &= 12 \text{ H}_{\text{MC}}, I_1 = 15 \text{ KGM}^2, \\ m_2 &= 17 \text{ K}\Gamma, m_3 = 35 \text{ K}\Gamma, m_4 = 27 \text{ K}\Gamma, \\ m_5 &= 7 \text{ K}\Gamma, R_1 = 36 \text{ CM}, r_1 = 25 \text{ CM}, \\ R_3 &= 26 \text{ CM}, R_4 = 20 \text{ CM}, r_4 = 12 \text{ CM}, i_4 = 16 \text{ CM}, \\ r_5 &= 12 \text{ CM}. \end{aligned}$$

Задача D-31.21.

Щеглов Иван



$$M_{D_z} = M_0 - k\omega_{3z}, \quad M_{n_z} = -\mu\omega_{4z},$$

$$M_0 = 10 \text{ Нм}, \quad k = 12 \text{ Н/м},$$

$$\varphi_{1,0} = 1.3, \quad \omega_{1z,0} = 0.2 \frac{1}{\text{с}},$$

$$\mu = 12 \text{ Нм}, \quad I_1 = 9 \text{ кгм}^2,$$

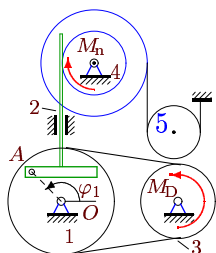
$$m_2 = 16 \text{ кг}, \quad m_3 = 34 \text{ кг}, \quad m_4 = 26 \text{ кг},$$

$$m_5 = 4 \text{ кг}, \quad R_1 = 35 \text{ см}, \quad r_1 = 24 \text{ см},$$

$$R_3 = 25 \text{ см}, \quad R_4 = 20 \text{ см}, \quad r_4 = 12 \text{ см}, \quad i_4 = 15 \text{ см}.$$

Задача D-31.22.

Карпов Юрий



$$M_{D_z} = M_0 - k\omega_{3z}, \quad M_{n_z} = -\mu\omega_{4z},$$

$$M_0 = 9 \text{ Нм}, \quad k = 12 \text{ Н/м},$$

$$\varphi_{1,0} = 1.2, \quad \omega_{1z,0} = 0.2 \frac{1}{\text{с}},$$

$$\mu = 13 \text{ Нм}, \quad I_1 = 7 \text{ кгм}^2,$$

$$m_2 = 15 \text{ кг}, \quad m_3 = 33 \text{ кг}, \quad m_4 = 25 \text{ кг},$$

$$m_5 = 3 \text{ кг}, \quad R_1 = 35 \text{ см}, \quad r_1 = 24 \text{ см},$$

$$R_3 = 25 \text{ см}, \quad R_4 = 20 \text{ см}, \quad r_4 = 12 \text{ см}, \quad i_4 = 14 \text{ см},$$

$$r_5 = 11 \text{ см}.$$