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Calculation of deformations of a cantilever-frame planar truss model with an arbitrary number of panels

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ABSTRACT

Introduction. By method of induction using three independent parameters (numbers of panels) formulas for deflection under different types of loading are derived. Curves based on the derived formulas are analyzed, and the asymptotic of solutions for the number of panels are sought. The frame is statically definable, symmetrical, with descending braces. The problem of deflection under the action of a load evenly distributed over the nodes of the upper chord, a concentrated load in the middle of the span, and the problem of shifting the mobile support is considered.

Materials and methods. The calculation of forces in the truss bars is performed in symbolic form using the method of cutting nodes and operators of the Maple computer mathematics system. The deflection is determined by the Maxwell – Mohr formula. Operators of the Maple computer mathematics system are used for composing and solving homogeneous linear recurrent equations that satisfy sequences of coefficients of the required dependencies. The stiffness of all truss bars is assumed to be the same.

Results. All the obtained dependencies have a polynomial form for the number of panels. To illustrate the obtained solutions and their qualitative analysis, curves of the deflection dependence on the number of panels are constructed.

Conclusions. A scheme of a statically definable three-parameter truss is proposed that allows an analytical solution of the problem of deflection and displacement of the support. The obtained dependences can be used in engineering practice in problems of structural rigidity optimization and for evaluating the accuracy of numerical solutions.

KEYWORDS: planar truss, frame, deflection, induction, Maple, analytical solution, console

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Расчет деформаций модели плоской фермы консольно-рамного типа с произвольным числом панелей

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АННОТАЦИЯ

Введение. Методом индукции по трем независимым параметрам (числам панелей), характеризующим пропорции конструкции и ее частей, дается вывод формул для прогиба при различных типах нагружения. Анализируются кривые, построенные по выведенным формулам, разыскивается асимптотика решений по числу панелей. Рама — статически определимая, симметричная, с нисходящими раскосами в простой решетке ригеля и консолей. Конструкция имеет подвижную и неподвижную опоры. Рассмотрены: задача о прогибе под действием нагрузки, равномерно распределенной по узлам верхнего пояса, сосредоточенной нагрузки в середине пролета и задача о смещении подвижной опоры.

Материалы и методы. Расчет усилий в стержнях фермы произведен в символьной форме с использованием метода вырезания узлов и операторов системы компьютерной математики Maple. Прогиб определен по формуле Максвелла – Мора с учетом только сжимающих и растягивающих усилий в стержнях. По результатам последовательности аналитических расчетов ферм с различным числом панелей в ригеле, консолях и опорных фермах методом индукции выведены итоговые расчетные формулы для прогиба и смещения опоры. Операторы специального пакета genfunc системы компьютерной математики Maple использованы для составления и решения однородных линейных рекуррентных уравнений, которым удовлетворяют последовательности коэффициентов искомых зависимостей. Жесткость всех стержней фермы принимается одинаковой.

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Результаты. Все полученные зависимости имеют полиномиальную по числу панелей форму. Для иллюстрации полученных решений и их качественноого анализа построены кривые зависимости прогиба от числа панелей.
Выводы. Предложена схема статически определимой трехпараметрической фермы, допускающая аналитическое решение задачи о прогибе и смещении опоры. Полученные зависимости могут быть использованы в инженерной практике в задачах оптимизации конструкции по жесткости и для оценки точности численных решений.

КЛЮЧЕВЫЕ СЛОВА: плоская ферма, рама, прогиб, индукция, Maple, аналитическое решение, консоль

ДЛЯ ЦИТИРОВАНИЯ: *Бука-Вайваде К., Кирсанов М.Н., Сердюк Д.О.* Calculation of deformations of a cantilever-frame planar truss model with an arbitrary number of panels // Вестник МГСУ. 2020. Т. 15. Вып. 4. С. 510–517. DOI: 10.22227/1997-0935.2020.4.510-517

INTRODUCTION

Planar trusses, as a rule, are the supporting components of the frames of spatial structures. Ignoring the work of links[1], the calculation of the spatial truss can be replaced by the calculation of individual planar trusses. Deflection calculation, the most difficult part of the design calculation in a practical sense, is usually performed numerically in known numerical packages based on the finite element method, including taking into account the nonlinearity of the problem [1–6] and dynamics. Accurate solutions in the form of simple formulas that depend on all design parameters are always a good help for such calculations. The more parameters are included in analytical solutions, the more valuable such formulas are. For example, the Kachurin's formula is widely known [7]. Formulas for some planar trusses with an arbitrary number of panels were obtained by V.A. Ignatiev [8]. The most effective method was an inductive method for obtaining formulas for deflection taking into account the number of panels. The monograph [9] provides more than 70 exact solutions to the problem of deformation of various statically definable flat trusses of beam, arch, and frame types obtained by induction using one or two integer parameters that characterize the number of panels in the structure. In [9] did not include (or did not include completely) the works [10–16], which also use the induction method for obtaining solutions in symbolic form. In [17, 18], the double induction method is used to derive the deflection formula. An overview of some works on this topic can be found in [19].

The main aim of this paper is to derive a formula for the deflection of a frame-type truss with three independent parameters that characterize the number of panels. A cantilever frame scheme is proposed (Fig. 1). A truss is a regular one to which the inductive method is applicable. For the first time, the problem of finding (“hunting”) statically definable regular trusses was raised by R.G. Hutchinson and N.A. Fleck [20, 21].

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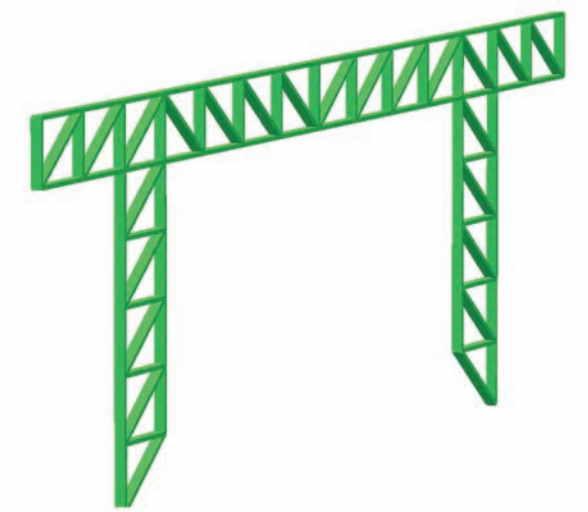

Fig. 1. Frame, $m = 5$, $n = 4$, $k = 2$

General problems of periodic bar structures are described in [22]. Analytical solutions can be used to solve structural optimization problems [23–29].

In this paper, we consider a truss consisting of a crossbar with $2n$ panels, two consoles with k panels in each, and support trusses with m panels in height. Together with three rigid bars that model supports, the truss contains $n_s = 8(n+m+k)+5$ bars.

MATERIALS AND METHODS

Calculating the vertical offset of node C (Fig. 2) performed using the Maxwell–Mohr's formula

$$\Delta = P\sum_{j=1}^{n_s-3}\frac{S_j s_j l_j}{EF}, \qquad (1)$$

where l_j and S_j is the length and force in the j-th bar from the action of the load, s_j — is the force from the unit force applied to the Central node C in the direction of the desired movement, E is the elastic modulus of the bars, F is the cross-sectional area.

Summation is performed on all the frame bars, except for the three support ones, which are accepted

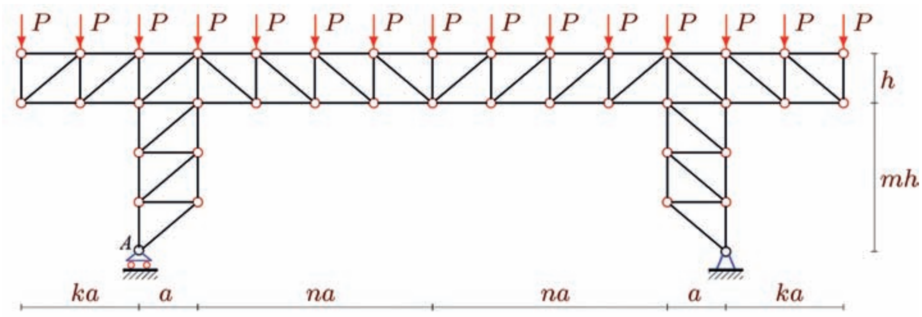

Fig. 2. The scheme of the truss, the load on the top chord, $m = 3$, $n = 4$, $k = 2$

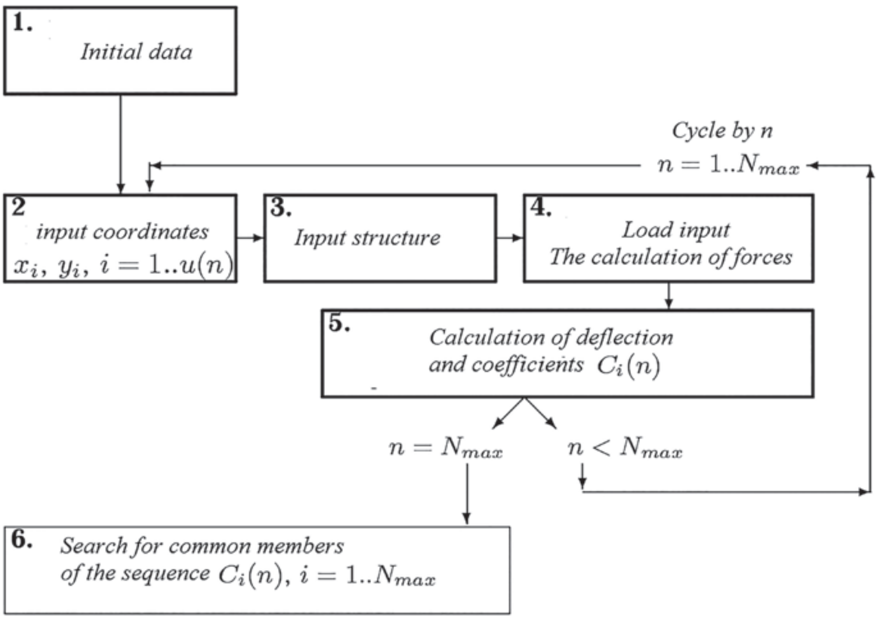

Fig. 3. The block diagram of the program

as non-deformable. The solution is searched for in an analytical form, so the efforts in the bars included in (1) must also be searched for in the form of formulas. The truss is statically definable, so it is most convenient to find it from the General system of equilibrium equations for all nodes (internal hinges). The coefficients in this system are the guiding cosines of the forces. The calculation is performed using the program [17, 18], written in the Maple language (Fig. 3).

At the beginning (block **1**), the program enters the estimated number of cycles N_{max}, which is sufficient for the program to allocate common terms of the sequence of coefficients of the desired formula for deflection. The dimensions of the structure and the load value are not set, these values are undefined parameters of the problem. In block **2**, the coordinates of the hinges are set. Entering coordinates for arbitrary parameters *m*, *n*, *k* is performed in cycles. In this problem, the hinges are numbered first along the inner and then along the outer contour of the truss (Fig. 4). Here is a fragment of the program with a set of coordinates, for example, the hinges of the left truss-rack:

```
> x[1]:=k*a:y[1]:=0:
> for i to m do x[i+1]:=k*a+a; y[i+1]:=h*i; end;
```

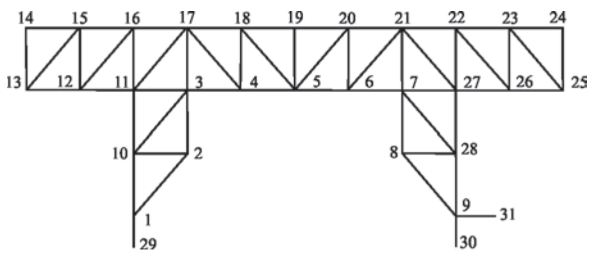

Fig. 4. The scheme of truss, $m = n = k = 2$

The structure of the connection of nodes (hinges) and bars in block **3** is set by special vectors containing the numbers of the ends of the bars. Vectors for entering bars along the internal and external contour of the truss have the form:

```
> m1:=m+n:
> for i to 2*m1 do
      N[i]:=[i, i+1]; end:
> for i to 2*m1+4*k+2 do
      N[i+2*m1]:=[i+2*m1+1, i+2*m1+2]; end:
```

Here **N** is a conditional vector with the number of the corresponding bar and coordinates equal to the numbers of nodes at its ends. The effort values do not depend on the choice of the directions of these vectors. In block **4**, the vector **B** of the right part of the system of equilibrium equations **GS** = **B** is created, where **G** is the matrix of the system's coefficients, and **S** is the vector of forces in all the truss bars, including the three reference ones. A uniform load on the upper chord nodes is introduced into the even elements of the vector:

> for i from 2*m1+m+k+2 to 3*m1+3*k+n+4 do B[2*i]:=1: end:

The matrix **G** of the node equilibrium equations is formed from the guiding cosines of the forces determined based on the specified geometry of the structure and the order of connecting bars. Odd rows of the matrix correspond to the projection of forces on the x axis, even-projections on the y axis, in the same block, the system of equations is solved. The inverse matrix method is used. In the Maple system, this is surprisingly simple. Maple works with matrices as with numbers:

G1:=1/G: S:=G1.B

Here **G1** is the inverse matrix. The matrix is multiplied by a column using the dot symbol.

In block **5**, using the Maxwell-Mohr's formula (1), an expression for deflection is found. It turns out that the type of solution does not change for trusses with different numbers n and m (a consequence of the regularity property of the construction):

$$\Delta = P\left(C_1a^3 + C_2c^3 + C_3h^3\right)/\left(h^2EF\right), \qquad (2)$$

where $c = \sqrt{a^2 + h^2}$ is the length of the brace. Elements of coefficient sequences are highlighted by the **coeff** operator.

Block **6**, which searches for common sequence members, is executed after the completion of a cycle of n deflection calculations. There is one problem here. If the cycle is not long enough, then the operator for detecting the recurrent equation for sequence elements does not give a plausible answer. In this case, you should increase $N_{\max}$ and continue the cycle. According to the operation condition of the **rgf_findrecur** operator for composing recurrent equations, the length of the sequence under study must be even. Block **6** gives an answer for solving the problem with random n, but fixed values of m and k. To get a formula that is valid for any n, m, k, you need to create two more external cycles for m and k, thus performing a triple induction. The need for triple induction is the main problem fot this truss, due to the fact that the maple system performs character transformations very slowly, especially for large-size matrices.

RESULTS

In the first of three cycles, the cycle for the number of panels n, the program outputs a sequence of coefficients before the cubes a^3, c^3 and h^3. For example, for a coefficient C_1 with $m = k = 1$, we have numbers: 3, 51/2, 92, 475/2, 507, 1911/2,... . The **rgf_findrecur** operator gives the following equation

$$C_{1,n} = 5C_{1,n-1} - 10C_{1,n-2} + 10C_{1,n-3} - 5C_{1,n-4} + C_{1,n-5}.$$

Its solution has the form

$$C_1 = (n+1)^2(5n^2 + 10n - 6)/12.$$

The calculation for m = 2, 3, 4 in the second cycle shows that this coefficient does not change. In the k loop we get

$$k = 2: \ C_1 = (n+1)^2(5n^2 + 10n - 30)/12,$$
$$k = 3: \ C_1 = (n+1)^2(5n^2 + 10n - 66)/12,$$
$$k = 4: \ C_1 = (n+1)^2(5n^2 + 10n - 114)/12,$$
$$k = 5: \ C_1 = (n+1)^2(5n^2 + 10n - 174)/12,$$
$$\ldots$$

Generalization by k only concerns the last term in these expressions. As a result, for random n, m, k we get

$$C_1 = (n+1)^2(5n^2 + 10n - 6k^2 - 6k + 6)/12. \qquad (3)$$

Similarly, for two other coefficients in (2) we get

$$C_2 = (n+1)^2/2,$$
$$C_3 = (n^2 + 2mn + 3m + 2km - 1)/2. \qquad (4)$$

The proposed formula output algorithm can be easily adapted to other loads. In the case of a load applied to the middle of the span to the node of the lower chord, the right part of the system of equations has the form

i: = m + n + 1: B[2*i]: = 1

Obviously, the consoles remain unloaded, and the number of panels k does not affect the deflection in any way. Induction over n and m gives the following expressions for coefficients in (2):

$$C_1 = (n+1)(2n^2 + 4n + 3)/6,$$
$$C_2 = (n+1)/2, \qquad (5)$$
$$C_3 = (n+m-1)/2.$$

The left support of the structure is movable and under the action of a vertical load it is displaced. The offset value is calculated in the same program:

$$\delta_A = P\left(A_1a^3 + A_2c^3 + A_3h^3\right)/\left(haEF\right),$$
$$A_1 = (4(1+2m)n^3 + 3(3+8m)n^2 +$$
$$+(5 - 6k^2 - 6k - 12mk^2 - 12mk + 22m)n -$$
$$-6m(k^2 + k - 1))/6, \qquad (6)$$
$$A_2 = m(2n+1),$$
$$A_3 = m(m+1)\left(2k + 2n + 3\right)/2.$$

Verification of the derived formulas can be performed either in numerical mode, or simply by changing the order of the parameters *m*, *n*, *k* in the induction process.

The obtained dependencies have a relatively simple form and are easily analyzed. Consider, for example, the case when the total number of panels in the crossbar and consoles is constant. Let $n + k = 20$. We also fix the span length $L = 2na$ = 100 m and the total load evenly distributed across the nodes of the upper chord $P_{sum} = P(2n + 2k + 2)$. The dependence of the dimensionless deflection $\Delta' = \Delta EF/(P_{sum}L)$ on the number of panels *n* (Fig. 5) shows that for small *n* and, consequently, large *k* (long consoles), the deflection

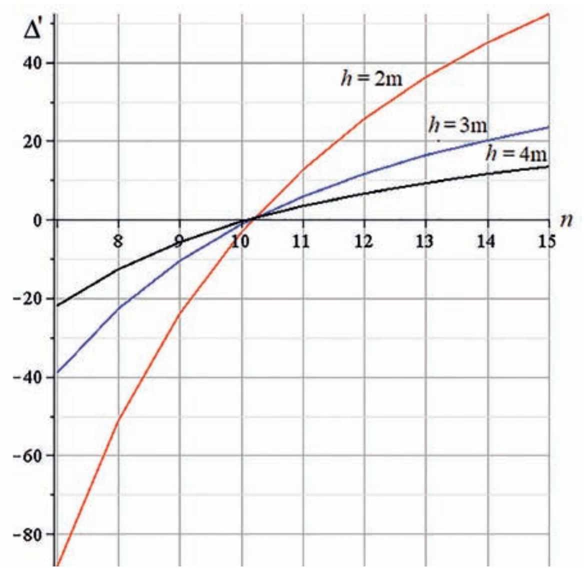

Fig. 5. Dependence of the deflection on the number of panels, $m = 3$, $L = 2na = 100$ m

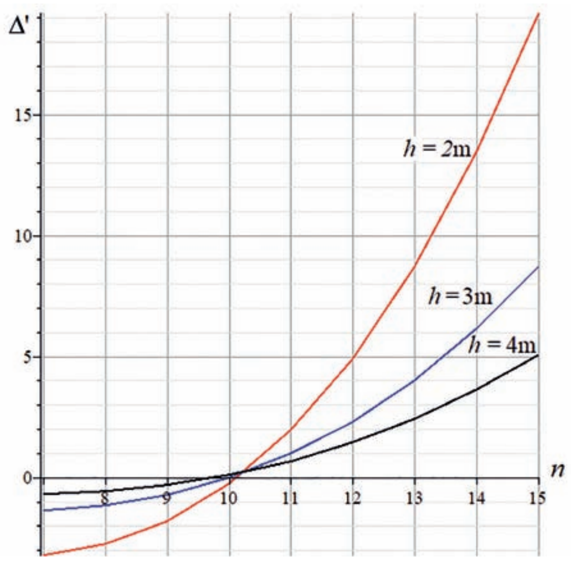

Fig. 6. Dependence of the deflection on the number of panels, $m = 3$, $L = 2a(n + k + 1) = 100$ m

is negative — the middle of the span is raised by the forces applied to the consoles. If the number n increases (the consoles are shortened), the crossbar in the middle naturally bends down. Almost the same effect is obtained, if you do not fix the span (the distance between the support of trusses), but the total length of the structure $L = 2a(n + k + 1) = 100$ m. The curves also have a common intersection point for different heights, but their convexity is directed downward (Fig. 6).

The analytical form of the solution using Maple methods allows us to find some of its asymptotics. For independent values of *n* and *m*, we have the following asymptotes of the solution (3, 4): $\lim_{n\to\infty} \Delta'/n = h/(2L)$, $\lim_{k\to\infty} \Delta'/k = -L^2(n+1)^2/(32n^3h^2)$. Note that if in the first case the slope of the asymptote increases with decreasing span length $L = 2na$, then in the second case (unlimited increase in console lengths) this dependence is reversed. The solution (6) of the problem of shifting the left movable support under the action of a vertical uniform load applied to the upper chord has similar asymptotes:

$$\lim_{n\to\infty} \delta'/n = h^2 m(m+3)/L^2,$$

$$\lim_{k\to\infty} \delta'/k = -L(n+m)/(8n^2h).$$

Here $\delta' = \delta EF/(P_{sum}L)$ is a dimensionless offset. When $\delta' > 0$, the support A (Fig. 2) is shifted to the left. The asymptotics of solving problem (6) for the number of panels *m* at a fixed height $H = mh$ is nonlinear:

$$\lim_{m\to\infty} \delta'/m^2 = \frac{L(2n+1)(2n^2+5n+6-3k^2-3k)}{24n^2(n+k+1)H}.$$

DISCUSSION AND CONCLUSIONS

The considered frame-type truss has a simple lattice for which it is easy to obtain internal forces values using standard methods, such as the cross-section method. The Ritter's cross section is available for almost all bars in the structure, except for only three bars above the truss supports. In spite of this, the method of cutting out nodes with the compilation of the matrix of the equilibrium equation of all nodes is used to get formulas for the deflection and displacement of the support. This is justified, firstly, by the fact that the reactions of the supports are determined simultaneously with the internal forces, and secondly, by using a computer program with free independent parameters (the number of panels), the result for the internal forces and, consequently, for the deflection is obtained automatically. This allows to get a series of solutions for which the desired formulas are derived by induction. For the solutions found, some asymptotics characteristics are revealed that give estimates of deflection or displacement with an extreme increase in the number of panels. As an illustration of the application of solutions, the paper presents a solution to the problem of the ratio of the number of panels in the crossbar and consoles. The characteristic point of intersection of curves corresponding to zero or very small deflection is found. Analytical solutions obtained by the induction method are convenient both for other, more complex problems of optimizing structures and for evaluating numerical solutions, especially since the accuracy of the analytical solution does not depend on the number of panels. For large-span structures, where the number of panels is very large, this is especially important.

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