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Mathematical model of a spatial rectangular contour-type truss deformations

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Abstract:

The object of research is spatial structure of a rectangular contour-type cover. A diagram of a statically determinate truss in the form of a closed rectangle with supports along the inner contour is proposed. The truss consists of quadrangular bar pyramids assembled into a square contour with tops connected by a bar belt. Four horizontal braces are located at the corners of the structure. A vertical load is considered, evenly distributed over the nodes of the truss. **Method.** The design is statically determinate, therefore, to calculate the forces in the rods, it is enough to solve the system of equations for the equilibrium of nodes. The matrix of the system of equilibrium equations consists of the direction cosines of the forces, which are calculated from the coordinates of the nodes. The derivation of the formula for the dependence of the deflection of several characteristic points of the structure on the number of panels in the truss is given. The conclusion is based on an inductive generalization of the decision sequence for structures with an increasing number of panels. The coefficients of the sought formulas are found from the solution of homogeneous linear recurrent equations. **Results**. The solution of the equilibrium equations of the nodes and all transformations are performed in the Maple symbolic mathematics system. Linear asymptotics of solutions are found. The two main results of the work are the development of a scheme for a regular spatial statically determinate rectangular truss and obtaining an analytical dependence of the deflection of the structure on the number of panels.

1 Introduction

Truss structures are most often used in roofing structures for public buildings, industrial and commercial enterprises. They are convenient and inexpensive to assemble, durable and versatile. Calculation of deformations, strength, and stability of truss structures is usually performed numerically in specialized computer programs based on the finite element method [1]–[3]. Numerical calculation is applicable for a very wide class of rod systems, including statically indeterminate ones. Analytical solutions used for simplified models of statically determinate structures are of particular importance in the calculations. The practical significance of such calculations is the greater, the more parameters of the object under study are included in the calculation formula, for example, the formula for the deflection or natural vibration frequency. In regular trusses containing periodic elements in their structure, it is most important to take into account their number (system order). This significantly expands the range of applicability of the formula and removes the question of accuracy, which is inevitable for large-order systems. The existence and design of statically determinate regular bar structures are considered in the works of Hutchinson and Fleck [4], [5]. Formulas for deflection of planar regular trusses, frames, and arches with various loads are collected in reference books [6], [7]. These formulas are obtained by induction in the Maple symbolic mathematics system [8]. The method of induction in the Maple system was applied in [9] to obtain a two-sided analytical estimate of the first oscillation frequency of a regular truss. In [10] the problem of the oscillation frequency of a two-span truss was solved using the Dunkerley method. Formulas for deflections of a planar arched truss of a regular type are obtained in [11], [12]. The formulas for the deflection of a planar externally statically indeterminate truss are obtained in [13].

In [14]–[16], to solve structural mechanics problems in an analytical form, the method of expanding solutions into series using the Maple symbolic mathematics system was applied. Algorithms for the analytical calculation of various types of lattices without using the induction method are given in [17], [18].

A new scheme for a regular statically determinate spatial coverage structure is proposed here. The task is to derive the analytical dependences of the truss deformations on the number of panels. The formulas obtained can be used to evaluate numerical solutions, especially for large-scale structures, for which an inevitable error of accumulated round-off errors appears in numerical calculations.

2 Materials and Methods

The goal was to develop a scheme of a regular statically determinate truss in the form of a square gallery with supports along the inner contour. The console part of the structure is supposed to accommodate objects around the entire perimeter, for example, cars that have free entry and exit. The gable shape of the coating is planned to protect the structure from atmospheric precipitation and increase the rigidity of the structure.

The proposed cover is a square structure in a plan, consisting of pyramids (Figure 1), connected at the tops with a rod square contour (Figure 2).

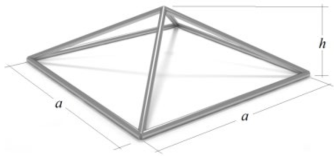

Fig. 1. Structural element (panel)

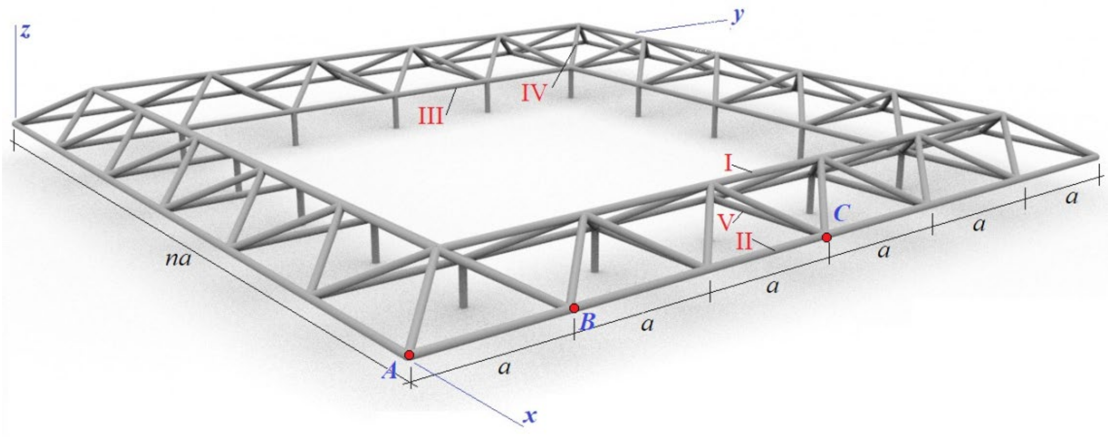

Fig. 2. Cover design and designation of some characteristic rods, *n*=6

The height of the pyramids *h*, the length of the side of the base *a*. The order $n>2$ of the regular construction is equal to the number of panels on each side of it. The lower faces of the pyramids form two contours. The inner lower contour of the structure is fixed on vertical posts and four additional horizontal ties at the corners of the structure (Fig. 3).

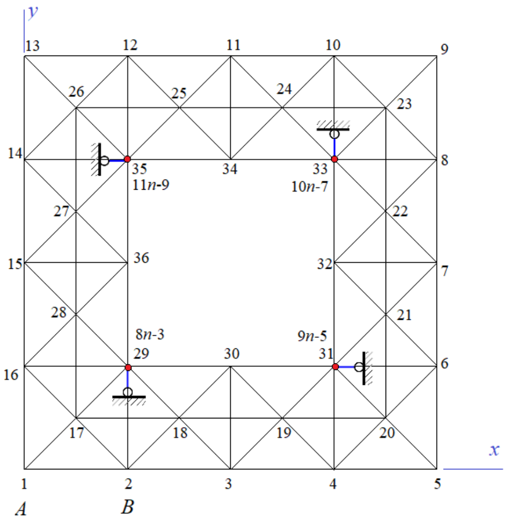

Fig. 3. Node numbering and horizontal links, *n*=4

The support rods are assumed to be non-deformable. The whole structure consists of $m = 36(n-1)$ rods including support rods. Uniform nodal vertical load is applied to all nodes, except for the nodes of the inner lower contour (Fig. 4). The outer lower contour of the truss forms a circular cantilever part of the structure.

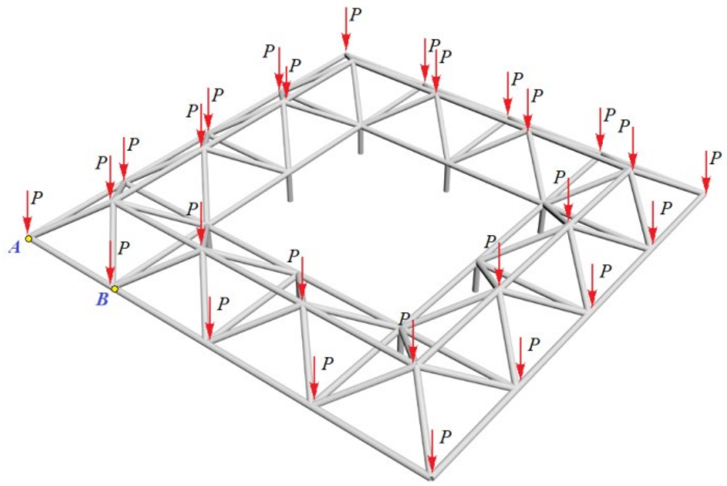

Fig. 4. Uniform load on the truss, *n* = 4. Horizontal ties in the inner corners of the structure are not shown

The absence of a diagonal tie in the base in the pyramids (Fig. 1) gives the structure additional volume.

The calculation of the forces in the rods is performed by cutting out nodes in an analytical form. To calculate the deflection, the Maxwell – Mohr formula is used under the assumption that all the rods, except for the support ones, are linearly elastic. In the sequences of coefficients in the formulas for trusses of various orders, there are common terms that give a view of the final dependence of deflections on the number of panels, truss size, and load. All transformations, solving the system of linear equations of equilibrium of nodes, and finding solutions to recurrent equations for the desired coefficients are performed in the Maple computer mathematics system.

3 Results and Discussion

3.1 Forces

The analytical form of the sought dependences of the structure deformations on its order involves the calculation of forces in a statically determinate structure in symbolic form. In the program written in the language of symbolic mathematics Maple [19], the equations of equilibrium of nodes in the projection on the coordinate axis are compiled. The system of equilibrium equations $G\overline{S} = \overline{T}$ ($\overline{S}$ —

vector of forces of length *k*, $\overline{T}$ — vector of loads) includes not only the forces in the rods but also the reactions of the supports. The nodes and bars of the truss are numbered (Fig. 3). The coordinates of the nodes of the lower outer contour, for example, have the form

$$\begin{aligned}
&x_i = a(i-1), y_i = 0,\\
&x_{i+n} = an, y_{i+n} = a(i-1),\\
&x_{i+2n} = a(n-i+1), y_{i+2n} = an,\\
&x_{i+3n} = 0,\ y_{i+3n} = a(n-i+1),\\
&z_i = z_{i+n} = z_{i+2n} = z_{i+3n} = 0,\ i = 1,...,n.
\end{aligned}$$

Coordinates of other nodes are set similarly (in a cycle). The order of connecting rods is similar to defining a graph in discrete mathematics. Bars are specified by their end numbers. The numbers are written in conditional vectors $N_i,\ i = 1,...,k.$ For example, for a closed lower outer contour: $N_i = [i, i+1],\ i = 1,...,4n-1,\ N_{4n} = [1, 4n].$

These data are used to calculate the direction cosines of the forces in the rods, which are elements of the matrix of the equations of the equilibrium of the nodes.

Equilibrium equations require the projection of the vector rods on the coordinate axes: $l_{x,i} = x_{N_{i,1}} - x_{N_{i,2}},\ l_{y,i} = y_{N_{i,1}} - y_{N_{i,2}}, l_{z,i} = z_{N_{i,1}} - z_{N_{i,2}}$. The direction cosine matrix G has components:

$$\begin{aligned}
&G_{3N_{i,1}-2,i} = l_{x,i} / l_i,\ G_{3N_{i,1}-1,i} = l_{y,i} / l_i,\ \ G_{3N_{i,1},i} = l_{z,i} / l_i,\\
&G_{3N_{i,2}-2,i} = -l_{x,i} / l_i, G_{3N_{i,2}-1,i} = -l_{y,i} / l_i,\ G_{3N_{i,2},i} = -l_{z,i} / l_i.
\end{aligned}$$

where $l_i = \sqrt{l_{x,i}^2 + l_{y,i}^2 + l_{z,i}^2}$. The first coordinate of the vector N_i is designated $N_{i,1}$ (conditional start of the bar), the second coordinate $N_{i,2}$ (end of the bar). Three rows of the matrix correspond to each node of the structure. The first two contain the direction cosines with the x and y axes, the third — with the vertical z axis.

On the right side of the system of equations, there are loads applied to the nodes: $T_{3i} = -P,\ i = 1,...,8n-4.$ The solution of the system of equations $G\overline{S} = \overline{T}$ gives the forces in the rods [22].

The picture of the distribution of forces in the truss rods with a load along the upper chord referred to the load P in the numerical form at $n = 4,\ a = h = 1\,\text{m}$, is shown in Fig. 5. The thickness lines are conditionally proportional to the modules of the corresponding forces. Stretched elements, for example, the bars of the upper contour, are highlighted in blue and compressed in red. The number indicates the value of the relative forces with an accuracy of two digits. Under such a load, the inner lower contour is compressed, the outer one, except for the corner bars, is stretched. All vertical support rods are compressed (not shown). Horizontal corner braces (not shown in the figure) are not stressed in the absence of a horizontal load.

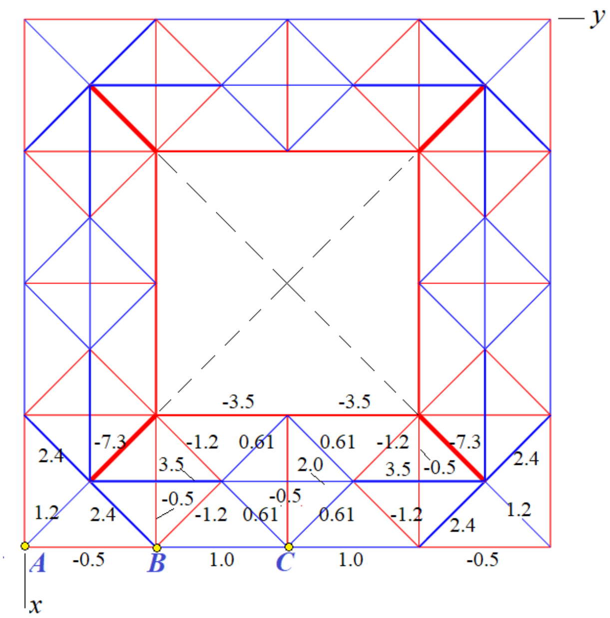

Fig. 5. Distribution of forces in the truss rods, *n*=4

Let us find analytical expressions for the dependencies in the most compressed and stretched bars of the structure. Consider the case of an even number of panels on the side of the structure $n = 2k$. Sequential calculation of the results in the bar of the middle side of the contour gives the following expressions

$$k = 2: \;\; S_{\mathrm{I}} = 2Pa / h;$$
$$k = 3: \;\; S_{\mathrm{I}} = -Pa / h;$$
$$k = 4: \;\; S_{\mathrm{I}} = -7Pa / h;$$
$$k = 5: \;\; S_{\mathrm{I}} = -16Pa / h, \ldots$$

The common term of this sequence gives an expression for the dependence of forces on the number of panels: $S_{\mathrm{I}} = -Pa(3k^2 - 9k + 2) / (2h)$. Similarly, formulas for forces in other rods are obtained:

$$S_{\mathrm{II}} = Pa(3k^2 - 3k - 2) / (4h), \;\; S_{\mathrm{III}} = Pa(3k^2 - 15k + 4) / (4h),$$
$$S_{\mathrm{IV}} = -3Pck / (2h), \;\; S_{\mathrm{V}} = -Pa / (2h).$$

Support reactions are found similarly (Fig. 6). By induction on the number of panels, the following expression is obtained: $Z' = (3n - 4)P, \; Z'' = -P.$

The reactions of the angular supports depend on the order of the system *n* and are directed upwards, while the values of the reactions of the internal supports do not depend on *n* and are directed downward, performing not a supporting, but a holding function.

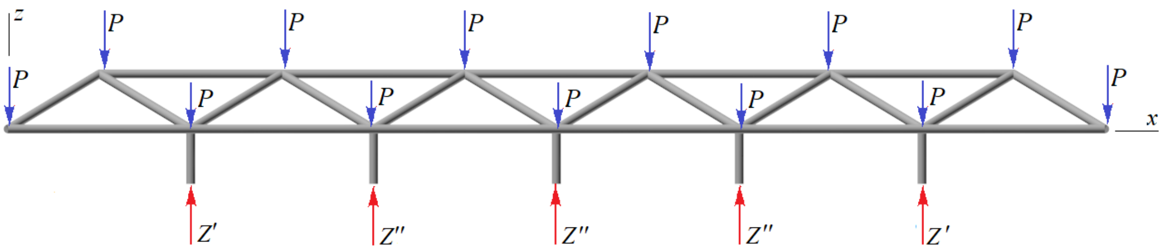

Fig. 6. Reactions of supports from the action of a uniform load, *n* = 6.

The graph (Fig. 7) shows the dependences of these forces, referred to the total load $P_0 = 4(4k - 1)P$ on the structure, assuming a constant length $L = 2ka = 10\,\mathrm{m}$ of the side of the truss.

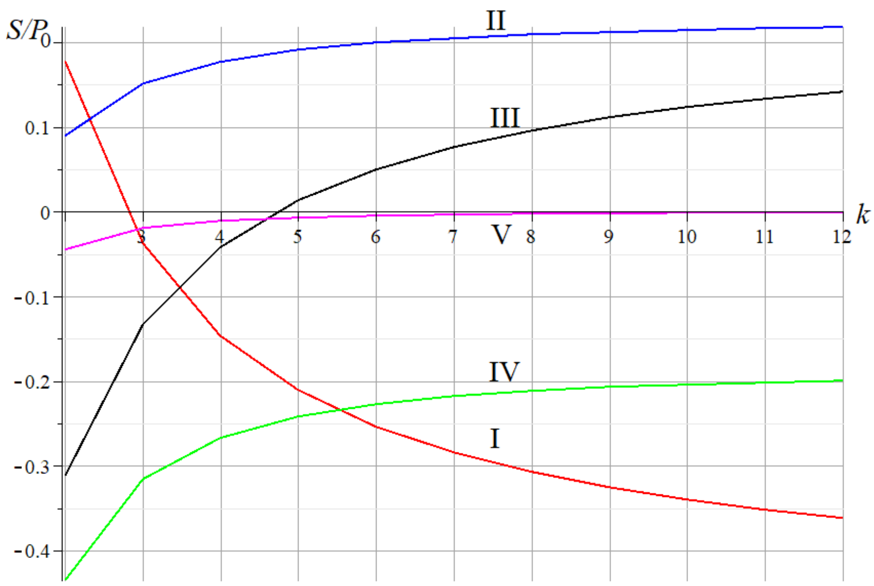

Fig. 7. Dependence of the relative forces in the rods on the number of panels

Depending on the number of panels, some of the bars can be either compressed or stretched. With an increase in the order of the structure *k*, the dependence of the relative forces in the proposed formulation (the length of the cover side and the total load are constant) levels out. At the beginning of the graph, with a small number of panels, the dependencies change quite significantly. The obvious asymptotics of the solutions can be traced:

$$\lim_{k\to\infty} S_{\mathrm{I}} / P_0 = -3L/(64h),$$

$$\lim_{k\to\infty} S_{\mathrm{II}} / P_0 = \lim_{k\to\infty} S_{\mathrm{III}} / P_0 = 3L/(128h),$$

$$\lim_{k\to\infty} S_{\mathrm{IV}} / P_0 = -3/16, \ \lim_{k\to\infty} S_{\mathrm{V}} / P_0 = 0.$$

For brace IV and horizontal connection V along the lower contours, the limiting values do not depend on the dimensions of the structure and are a kind of constants of the structure scheme.

3.2 Deflection

Select three points *A, B, C* on the outer bottom contour, the deflection of which will be calculated. In the case of calculating the deflection of node *C* the even order of truss is assumed: $n = 2k, k = 1,2,3...$ The deflection is determined by the Maxwell – Mohr formula:

$$\Delta = \sum_{\alpha=1}^{m-\tilde{m}} \frac{S_\alpha^{(P)} S_\alpha^{(1)} l_\alpha}{EF}. \tag{1}$$

The sum is calculated for all elastic bars of the structure, except $\tilde{m} = 4(n-1)$ for the supporting ones, which are assumed to be rigid. The following designations are introduced: $S_\alpha^{(P)}$ — force in a bar with a number from the action of an external load, $S_\alpha^{(1)}$ — force in the same bar from the action of a single vertical force applied to the node whose deflection is measured, l_α — bar length, *EF* — bar stiffness.

For the deflection of node *A* of a row of trusses with a sequentially increasing number of panels, the following formulas are obtained

$$n=3,\ \Delta=\frac{P(116a^3+11c^3)}{16h^2EF},$$
$$n=4,\ \Delta=\frac{P(96a^3+7c^3)}{8h^2EF},$$
$$n=5,\ \Delta=\frac{P(232a^3+17c^3)}{16h^2EF},$$
$$n=6,\ \Delta=\frac{5P(10a^3+c^3)}{4h^2EF},...$$

Thus, a general view of the dependence of the deflection on the number of panels is:

$$\Delta=\frac{P(C_1a^3+C_2c^3)}{h^2EF}, \quad (2)$$

where $c=\sqrt{2a^2+4h^2}$. The coefficients C_1 and C_2 in this dependence are determined by the induction method.

For example, the coefficient C_1 is found from the solution of the homogeneous linear recurrent equation of the ninth order

$$C_{1,n}=5C_{1,n-1}-10C_{1,n-2}+10C_{1,n-3}-5C_{1,n-4}+C_{1,n-5},$$

This equation gives the Maple operator rgf_findrecur for the values of the sequence of coefficients 116/16, 96/8, 232/16, 50/4, ... This operator has the following call form: **rgf_findrecur**(N, seq, u, n), where N is the order of the recurrent equation equal to the number of pairs of numbers in the sequence under study seq, u is the function name for sequence, *n* — name, index variable for the recurrence. For example, the operator **rgf_findrecur** (2, [1, 2, 4, 8], u, n) gives the equation u (n) = 2u (n-1). In cases where the length of the sequence is not enough to obtain the equation, the operator gives an equation with fractional coefficients, which does not make sense in this formulation. This forces you to do a few more (even number) truss calculations.

To solve recurrent equations, the Maple system has the rsolve operator. Taking into account the initial data $C_{1,3}=116/16,\ C_{1,4}=96/8,\ C_{1,5}=232/16,\ ...$, the solution is obtained in the form

$$C_1=-(n-1)(3n^2-24n+16)/16. \quad (3)$$

Similarly, find the coefficient

$$C_2=(3n+2)/16. \quad (4)$$

The formula for the deflection of node *B* has the same form (2), but with the coefficients

$$C_1=-(n-1)(3n^2-36n+40)/16,$$
$$C_2=(9n-4)/32. \quad (5)$$

The most difficult was the derivation of the formula for the deflection of the middle of the lateral side of the structure at point *C*. The case of an even number of panels was considered $n=2k$. The coefficients in (2) in this case have the form

$$C_1=(15k^4-90k^3+199k^2-144k+30)/8,$$
$$C_2=(6k^2-3k+4)/16. \quad (6)$$

Let us illustrate the dependence of the deflection on the number of panels for a truss of length $L=na=15\,\mathrm{m}$ with a total load $P_0=(8n-4)P$ on the lower chord. Let us introduce the designation for the dimensionless deflection: $\Delta'=\frac{EF\Delta}{P_0L}$. Figure 8 shows three curves constructed according to formulas (2-4) for the deflection of the corner node *A*. It is interesting to note that the corner of a truss with a small number of panels falls, and starting from a certain critical number of panels, depending on the height of the truss, the angle under the action of the load rises.

The found dependence has a limiting value in a non-horizontal asymptote. The limit can be found using the Maple system: $\lim_{n\to\infty} \Delta' = \frac{3h}{16L}$.

The limit value is positive; therefore, the curves cross the axis for some values $\tilde{n}$ of the number of panels. However, an approximate calculation shows that it is unrealistically large, and the effect of changing the sign of the deflection with a large number $\tilde{n}$ of panels has no practical significance.

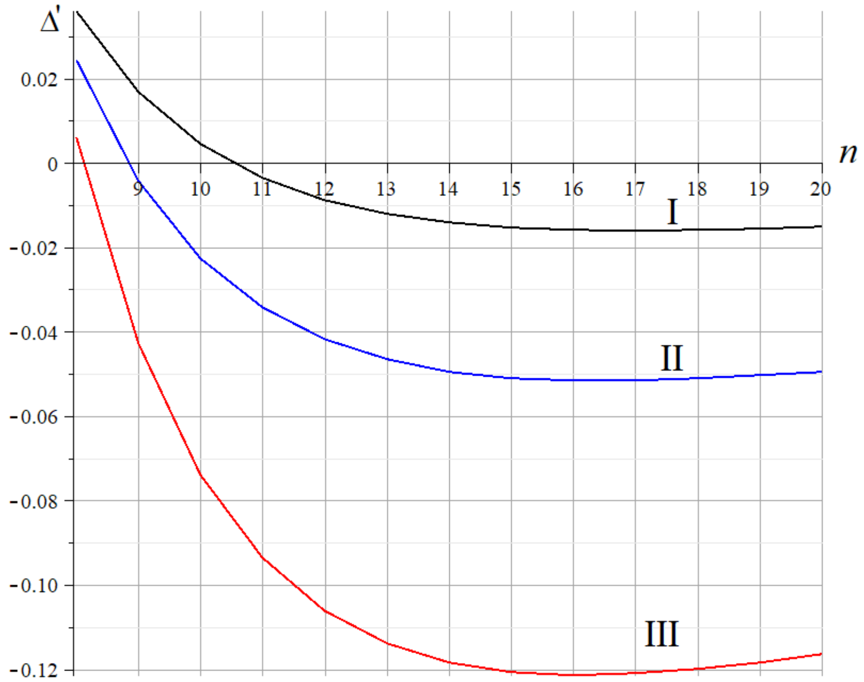

Fig. 8. Deflection of node *A* depending on the number of panels and the height of the truss, I — $h = 2.5\,\mathrm{m}$; II — $h = 2.0\,\mathrm{m}$; III — $h = 1.5\,\mathrm{m}$

A different picture (Fig. 9) of the dependence of the deflection on the number of panels is given by function (2) with the coefficients (5) calculated for the case of the deflection of node B. The calculation was performed under the same conditions as for node *A*, but for $L = 50\,\mathrm{m}$. The order of the curves plotted for different heights changes after a certain value of *n*. The node, instead of lowering $\Delta' > 0$, rises $\Delta' > 0$, in the direction opposite to the action of the load. In this case, the amount of upward movement is greater, the smaller the height of the truss *h*. There is also a horizontal asymptote here: $\lim_{n\to\infty} \Delta' = \frac{9h}{32L}$.

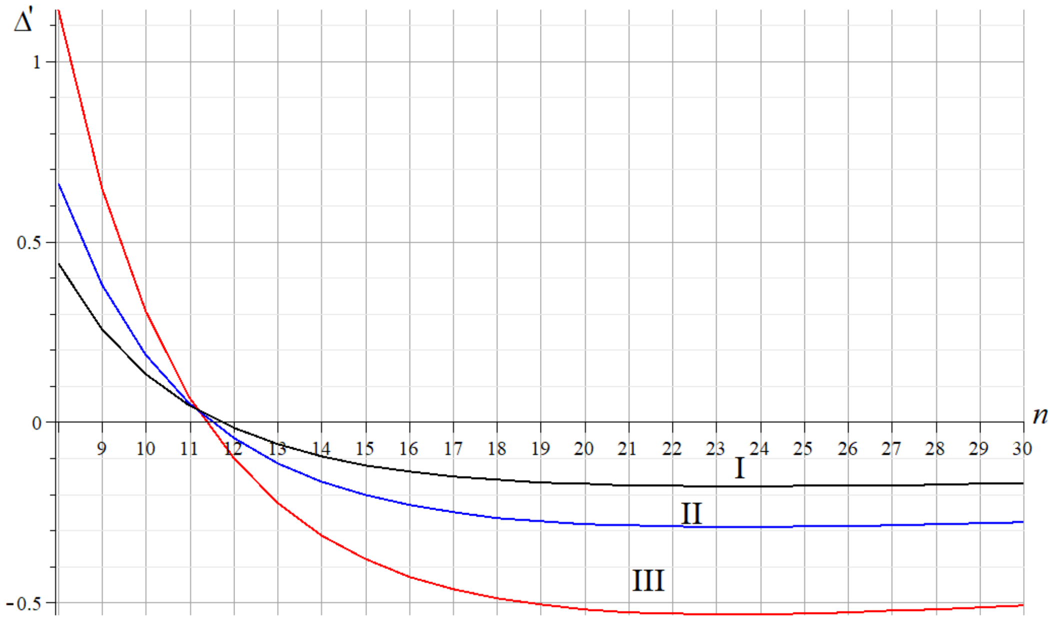

Fig. 9. Deflection of node *B* depending on the number of panels and the height of the truss, I — $h = 2.5\,\text{m}$; II — $h = 2.0\,\text{m}$; III — $h = 1.5\,\text{m}$

The deflection of the truss in the middle of the side (node *C*), calculated by the formula (2) with coefficients (6), increases monotonically with an increase in the number of panels at a fixed side length $L = na = 50\,\text{m}$ (Fig. 10).

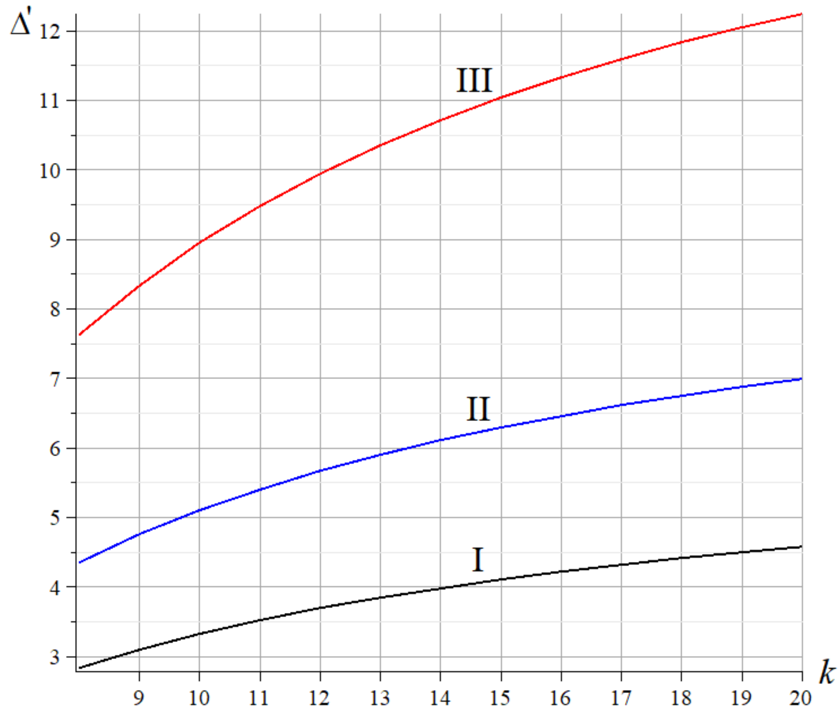

Fig. 10. Deflection of node *C* depending on the number of panels and the height of the truss, I — h = 2.5 m; II — h=2 m; III — h=1.5 m

Let's calculate the deflection from the action of an asymmetric load. Consider the action of a concentrated force in the middle of the span (*n* is an even number). In the calculation program, formula (1) will take the form

$$\Delta = P\sum_{\alpha=1}^{m-\tilde{m}} \left(S_{\alpha}^{(1)}\right)^2 l_{\alpha} / (EF).$$

The free member column will also change. Only one element of the vector on the right side of the system will be nonzero $T_{3i} = -P,\ i = k+1.$

For trusses with different numbers of panels, a sequence is obtained

$$k=3,\ \ \Delta = \frac{P(344a^3+21c^3)}{32h^2EF},$$
$$k=4,\ \ \Delta = \frac{P(4432a^3+129c^3)}{144h^2EF},$$
$$k=5,\ \ \Delta = \frac{P(4404a^3+73c^3)}{64h^2EF},$$
$$k=6,\ \ \Delta = \frac{P(10468a^3+111c^3)}{80h^2EF},...$$

The length of the sequence of solutions required to detect the pattern for an asymmetric load turned out to be 10. An additional difficulty in this calculation resulted from the fact that with an increase in the number of panels, not only the numerator in the answer changed, but also the denominator. Using the induction method from the results of analytical solutions, the following coefficients in (2) are obtained

$$C_1 = \frac{24k^4-96k^3+169k^2-131k+36}{24(k-1)},$$
$$C_2 = \frac{4k^2-6k+3}{16(k-1)}. \qquad (7)$$

Similarly (for arbitrary numbers *n*), the coefficients in (2) are obtained when calculating the deflection of the corner joint *A* from the action of the concentrated force *P*

$$C_1 = \frac{6n^4-21n^3+26n^2-17n+12}{6(n-2)(n-1)^2},$$
$$C_2 = \frac{4n^3-8n^2+9n+2}{16(n-2)(n-1)^2}. \qquad (8)$$

If a vertical force *P* is applied to the hinge *B*, then it receives a deflection calculated using the same formula (2) with the coefficients

$$C_1 = \frac{20n^4-108n^3+225n^2-221n+90}{24(n-2)(n-1)^2},$$
$$C_2 = \frac{2n^3-7n^2+7n-1}{8(n-2)(n-1)^2}. \qquad (9)$$

Note that for an asymmetric load, the solution is much more complicated even for one concentrated force. In the last three solutions, finding the common members of the sequences required analyzing the denominators of the sequence members. In the case of loading nodes *A* and *B*, the denominators nonlinearly depend on the number of panels. Maple operators for linear recurrence equations cannot find common members of such sequences. The type of the denominator here was selected manually.

The reaction to the action of force at the corner points *A, B* and in the middle of span *C* turns out to be completely different. This can be seen from the relative deflection curves plotted in Fig. 11. With an increase in the number of panels at a constant truss side length L, the deflection at corner point *A* and point *B* decreases monotonically. Moreover, there is a limit (horizontal asymptote) $\lim_{k\to\infty} \Delta'_A = \Delta'_B = 2h/L.$

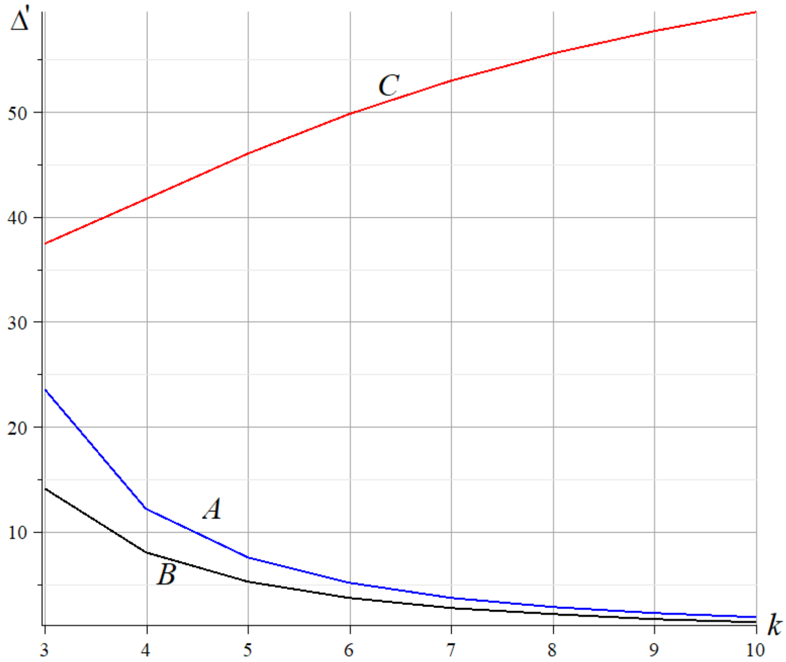

Fig. 11. Dependence of the deflection at points *A, B* and *C* on the number of panels under the action of concentrated forces on them. $L = 50\text{m}, h = 2\,\text{m}$

In the middle of the span (point *C*), the deflection increases with the number of panels. There is an oblique asymptote. In addition, from a comparison of these results with deflections from the action of a distributed load with the same dimensions of the structure, it follows that the deflections from a concentrated load are much larger.

Corner supports create constraints in the vertical direction and one horizontal direction. Consequently, the support can be displaced along a different horizontal axis. It is not difficult to calculate the dependence of this displacement on the number of panels. If a uniform vertical nodal load is considered, then in the Maxwell-Mohr formula (1) the force $S_\alpha^{(1)}$ is from the action of the horizontal force. For nodes $8n-3$ and $10n-7$, the unit force causing forces $S_\alpha^{(1)}$ is axially x directed (Fig. 3).

Using the induction method, an expression is obtained for the horizontal displacements of the supports

$$\delta_x = Pa^2(n-2)(n^2-13n+8)/(8hEF).$$

Figure 12 shows this dependence for the dimensionless quantity $\delta' = EF\delta/(P_0L)$ at $L = 50\text{m}$. As the number of panels increases, the horizontal offset of the corner point decreases and then changes sign. It is characteristic that the critical number of panels corresponding to the change of signs does not depend on the size of the truss and the load. When rounding the root $(13+\sqrt{137})/2$ of equation $\delta_x = 0$ to an integer, it turns out that the change of sign of the offset occurs at $n = 12$.

After this point, there is also a change in the order of the curves obtained for different values of the height h. The resulting dependence has a horizontal asymptote $\lim\limits_{n\to\infty} \delta_x' = -L/(64h)$.

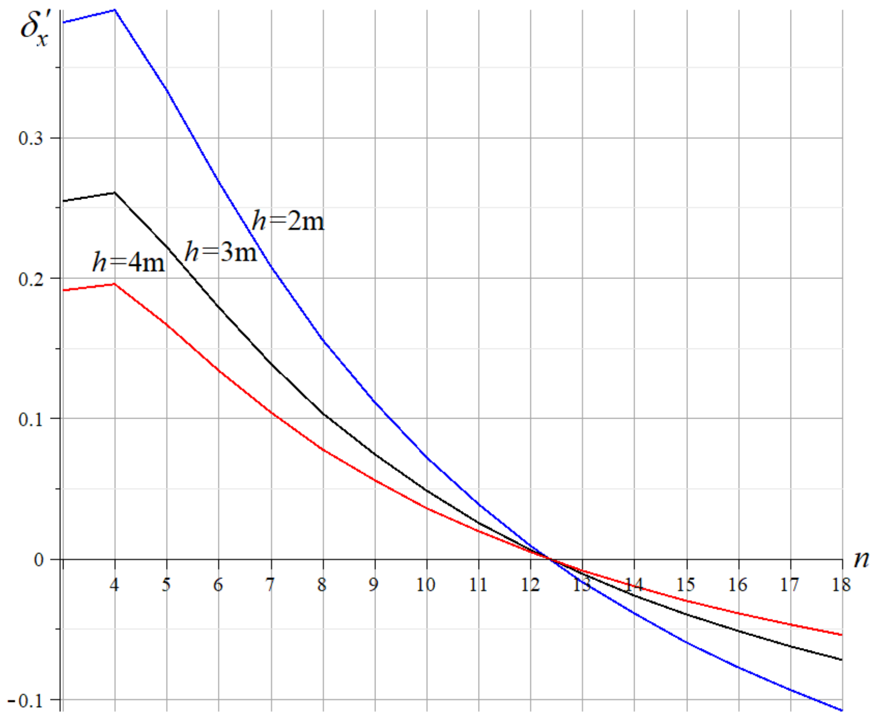

Fig. 12. Dependence of the horizontal displacement of the corner supports on the number of panels under the action of a distributed load, $L = 50\text{m}$

3.3 Discussion

The development of a statically definable spatial regular structure has specific features. For example, if the truss is rectangular, then four corner supports are assumed for it. This contradicts the condition that the plane is defined by three points. Of course, you can add fourth support and find its reaction by the force method or simply by choosing the reaction value from the symmetry of the problem and the equilibrium condition of the structure. But this approach deprives the scheme of universality to loads. For each load, especially an asymmetric one, it is required to select this value of this reaction or to re-disclose the static uncertainty. In the proposed coverage scheme, it was possible to obtain a solution to the bearing problem. Moreover, the proposed design has a cantilever part overhanging about the sides, which creates additional space not cluttered by supports. A regular triangular spatial truss [20] also calculated analytically, has a similar property. The tetrahedral bar pyramids were also used in the coverage scheme [21]. The lower limit of the first frequency of this truss was obtained analytically by the Dunkerley method [22].

The algorithm used to derive the formula for the deflection depending on the number of panels is known [19]. The peculiarity of using this approach for calculating the considered spatial structure manifested itself, as expected, firstly in the complication of compiling a mathematical model, and secondly, in a significant increase in the computation time. Symbolic conversions are noticeably more time-consuming than numeric conversions.

The proposed scheme and its calculation can be considered as some basic one for more complex statically indeterminate structures, for example, with the overlap of the inner "courtyard" formed by the gallery. Without much difficulty, the solution can be generalized to a non-square structure.

Note that the proposed solution is designed for rods of the same stiffness. However, the generalization of the solution to an arbitrary ratio of the stiffness of the rods is easy. In formula (2) it is necessary to enter the coefficients of relative stiffness:

$$\Delta = \frac{P(C_1 a^3 + \gamma C_2 c^3)}{h^2 EF}.$$

In this case, EF is the stiffness of horizontal bars of length *a*, and EF/γ is the stiffness of inclined braces of length *c*.

4 Conclusions

The proposed new scheme of a complex spatial cover and analytical expressions for its stress-strain state is obtained. Statically determinate schemes of spatial regular constructions are rare. Hutchinson, R.G., Fleck, N.A. [5] the search for such schemes was even called "hunting". The proposed scheme is of a regular type, and the cantilever shape of its gallery makes it possible to use such a structure in the organization of structures with free access to the premises from four sides. The designers may somehow complicate the structure by adding some additional rods, supports, and connections to it. In this case, the proposed calculation formulas can be used by designers as some estimate of deflections and forces in a complicated structure.

The main value of the solution obtained is the dependence on the number of panels. It is not difficult to obtain a simple formula for calculating the deflection of a specific structure. It is much more difficult and more important to find the dependence of the solution on the number of periodicity elements (panels, for example). Such solutions can be analyzed analytically and the most optimal parameters of the designed structure can be selected without volumetric recalculation of options in numerical form. Here, in the solution, asymptotics was found for the forces in some rods and deflections of the characteristic nodes of the truss.

Compared with the known solutions, the formulas obtained are quite simple and have the form of polynomials in the number of panels. One of the advantages of an analytical solution is the independence of its accuracy from the complexity of the design. For spatial trusses, this advantage is even more pronounced than for planar trusses.

In three-dimensional problems, with an increase in the number of rods in the structure, the order of the system of equations grows faster and the limit after which the error in numerical calculations becomes unacceptable is reached earlier.

In addition, analytical solutions have more options for analyzing them. In particular, based on analytical solutions, it is easier to solve the problems of optimizing structures both in terms of weight and strength, and stability.

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