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INDUCTIVE ANALYSIS OF A PLANAR TRUSS WITH A COMPLEX LATTICE

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The calculation of trusses in practice is usually done numerically in specialized packages based on the finite element method. In this case, the calculation can take into account the ways of connecting the rods, the heterogeneity of the material, calculate the dynamics and solve many other complex problems. The basis of any calculation of the trusses is the statically determinate model of the truss with the hinged junctions of the nodes. Analytic solutions of the problem of the deflection of a truss under the influence of an external load for trusses with an arbitrary number of panels of simple formula type are available for such regular-type periodic or almost periodic structures. Before the advent of systems of symbolic mathematics, such solutions were rare. For example, Kachurin's formula for a wide class of beam trusses [1] and Ignatiev's formula for a cantilever beam with a cruciform lattice are known [2]. Recently, exact analytical solutions have appeared for various planar [3–8] and spatial trusses [9], obtained by induction in the Maple system.

Consider a beam truss with a double lattice (fig. 1). The descending slopes have a slope of $h/(2a)$, the ascending slopes are h/a. The conditional number of panels n is counted according to the number of rods of length a in half span over the upper belt. In the truss, $m = 8n + 16$ rods together with three support rods and $k = 4n + 8$ hinges. The load is evenly distributed over the nodes of the lower belt.

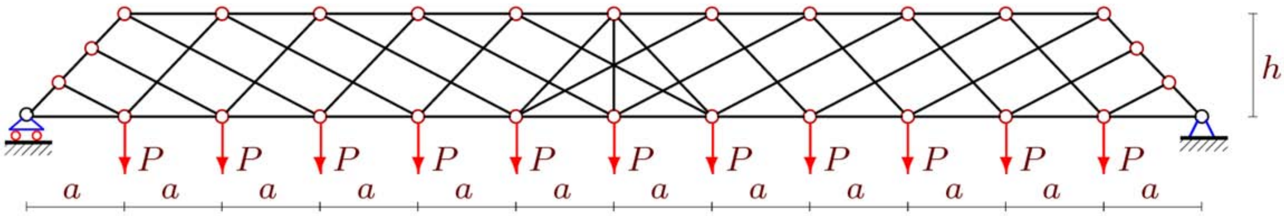

Fig. 1. The load on the bottom chord, $n = 5$

The task is to obtain an analytical dependence of the deflection of the truss on the number of panels. To calculate the forces in the rods, the program [8, 9] is used, which allows the method of cutting the nodes to find the forces in the rods in a symbolic form. The coordinates of the hinges are entered in the program (fig. 2). The origin is located in the left movable support.

The coordinates of the truss hinges with an arbitrary number of panels n have the form:

$$\begin{array}{lll} x_i = a(i-1), & y_i = 0, & i = 1, \ldots, 2n+3, \\ x_{i+2n+5} = ai, & y_{i+2n+5} = h, & i = 1, \ldots, 2n+1, \\ x_{i+2n+3} = ai/3, & y_{i+2n+3} = hi/3, & \\ x_{i+4n+6} = x_{4n+6} + ai/3, & y_{i+4n+6} = h - hi/3, & i = 1, 2. \end{array}$$

The order in which the bars are connected is defined in the program by lists that contain the numbers of the ends of the bars. The lower belt rods are inserted in the cycle:

$$N_i = [i, i+1], \qquad i = 1, \ldots, 2n+2.$$

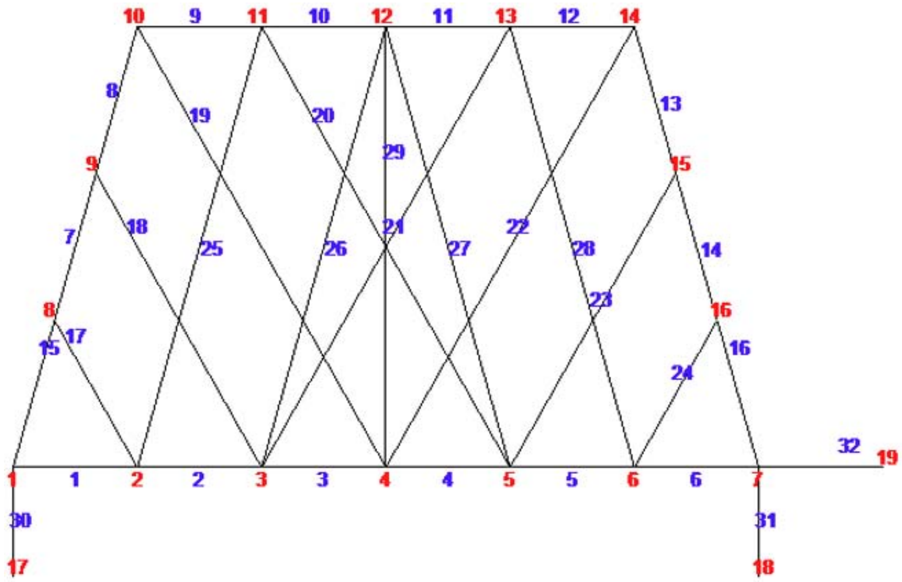

Fig. 2. The numbering of the rods and nodes in the program, $n = 2$

The side and top belt bars are coded as follows:

$$N_{i+2n+2} = [i + 2n + 3, i + 2n + 4], \quad i = 1, \dots, 2n + 4,$$
$$N_{4n+7} = [1, 2n + 4], \quad N_{4n+8} = [2n + 3, 4n + 8].$$

For grid bars, the number lists are as follows

$$N_{i+4n+8} = [i + 1, i + 2n + 3], \quad N_{i+5n+10} = [i + n, i + 3n + 6], \qquad i = 1, \dots, n + 2,$$
$$N_{i+6n+12} = [i + 1, i + 2n + 6], \quad N_{i+7n+12} = [i + 3n + 5, i + 2 + n], \quad i = 1, \dots, n.$$

The middle stand has a code:

$$N_{8n+13} = [n + 2, 3n + 6].$$

For rods supports written lists

$$N_{m-2} = [1, k + 1], \quad N_{m-1} = [2n + 3, k + 2], \quad N_m = [2n + 3, k + 3].$$

Three hinges with coordinates

$$x_{k+1} = 0, \qquad y_{k+1} = -b,$$
$$x_{k+2} = x_{2n+3}, \qquad y_{k+2} = -b,$$
$$x_{k+3} = x_{2n+3} + b, \quad y_{k+3} = 0$$

are support, the equilibrium of these nodes is not considered, the size b can be arbitrary, since the condition of the support rods are not deformed and this size is not included in the solution. The coordinates and the numbers of ends of the rods are calculated, the guides of the cosines of the stresses in the bars that are made to the matrix of equilibrium equations [7–9]. Analytical expressions for forces are obtained from the solution of the system of equations.

Deflection is calculated using the Mohr's integral:

$$\Delta = \sum_{i=1}^{m-3} \frac{S_i^{(P)} S_i^{(1)} l_i}{EF}.$$

Here is denoted: $S_i^{(P)}$ — the forces in the rods from the given load, l_i — the length of the rod, $S_i^{(1)}$ — the force from the unit force applied to the knot of the lower belt in the middle of the span, EF — the stiffness of the rods. The solution has the form

$$\Delta_n = \frac{C_1 a^3 + C_2 c^3 + C_3 f^3 + C_4 h^3}{2h^2 EF},$$

where $c = \sqrt{a^2 + h^2}$, $f = \sqrt{4a^2 + h^2}$ and the coefficients depend only on the number of panels by the induction method are determined. In any case, for arbitrary trusses, the type of solution depends on the number of panels. For example, for trapezoidal trusses or trusses with a triangular outline of the upper belt, with an increase in the number of panels, the number of terms in the expression for deflection also increases. The regularity property, connected as a consequence with the independence of the general form of the expression for the deflection from the number of panels, allows one to obtain a generalization of the solution to the case of an arbitrary number of panels. This significantly expands the scope of the final formula.

Based on the results of the successive calculation of the trusses with $n = 1, 2, \ldots$, a sequence of coefficients is found at a^3, c^3, f^3 and h^3, of the length at which the patterns of their formation can be determined. The Maple operator rgf_findrecur allows you to obtain a recursive equation for the common term of the sequence, and the rsolve operator gives a solution to this equation. For the coefficient at a^3, as a result of the solution of the problem for 18 trusses with the number of panels from 1 to 18 in half of the span, a sequence of numbers 30, 83, 246, 591, ..., 70275, 88158, 109381 was obtained. The recursion equation of the ninth order for this coefficient has view

$$C_{1,n} = 3C_{1,n-1} - 3C_{1,n-2} + 3C_{1,n-3} - 6C_{1,n-4} + 6C_{1,n-5} - 3C_{1,n-6} + 3C_{1,n-7} - 3C_{1,n-8} + C_{1,n-9}.$$

The solution of this equation is:

$$C_1 = \frac{15n^4 + 60n^3 + 129n^2 + (24p_n - 8q_n + 146)n + 40p_n - 48q_n + 66}{18},$$

where $p_n = \sqrt{3}\sin(2n\pi/3)$, $q_n = \sin(2n\pi/3)$ are periodic functions of n.

Similarly, from the solution of equation

$$C_{2,n} = C_{2,n-1} + 2C_{2,n-3} - 2C_{2,n-4} - C_{2,n-6} + C_{2,n-7}$$

we have the coefficient at c^3

$$C_2 = \frac{15n^2 + 2(5p_n - 3q_n + 21)n + 6p_n + 2q_n + 25}{27}.$$

From the solution of the same equation, but with other initial conditions, the coefficient C_3 is obtained

$$C_3 = \frac{15n^2 + 2(2p_n - 6q_n + 15)n + 3p_n - 13q_n + 13}{27}.$$

The coefficient at h^3 is obtained as a solution to a simpler equation: $C_{4,n} = 2C_{4,n-3} - C_{4,n-6}$ and has the form

$$C_4 = \frac{2(2(p_n + q_n + 2)n + p_n + 5q_n + 4)}{9}.$$

Example. For a truss with a span of $L = 2(n+1)a = 30$ m, we plot the dependence obtained for different values of the height h. We introduce the relative deflection $\Delta' = \Delta EF/(P_s L)$, where $P_s = (2n+1)/P$ is the total load on the truss. The nonmonotonic nature of the dependence (fig. 3) assumes the existence of a minimum. A limit

$$\lim_{n\to\infty} \frac{\Delta'}{n} = \frac{5h}{18L} > 0$$

indicating the presence of an increasing asymptote proves that a decrease in deflection after a certain value of n will be replaced by its growth.

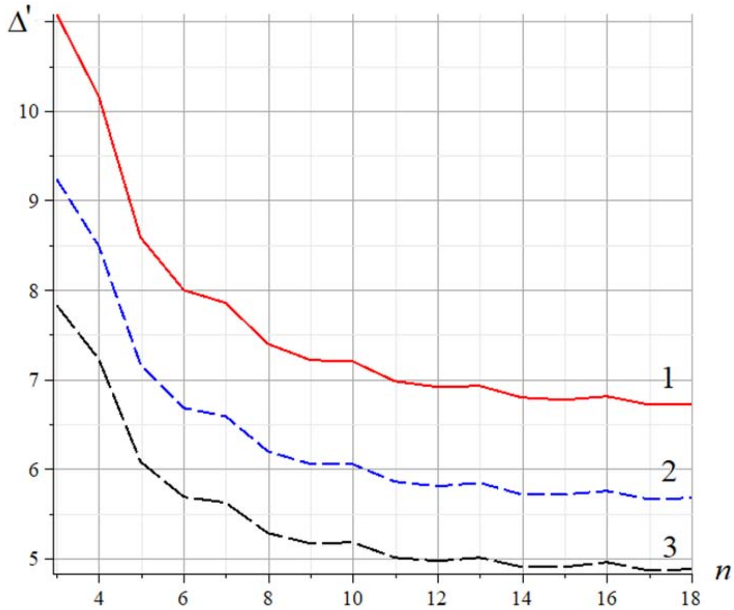

Fig. 3. Dependence of the relative deflection on the number of panels.
$1 - h = 2.0$ m; $2 - h = 2.2$ m; $1 - h = 2.4$ m

By means of Maple it is also possible to show character of distribution of forces in rods of a truss (fig. 4). The forces in the rods are proportional to the thickness of the depicting segments of the rods. It can be seen that the main load is perceived by the belts, and the most compressed struts are in the middle of the span.

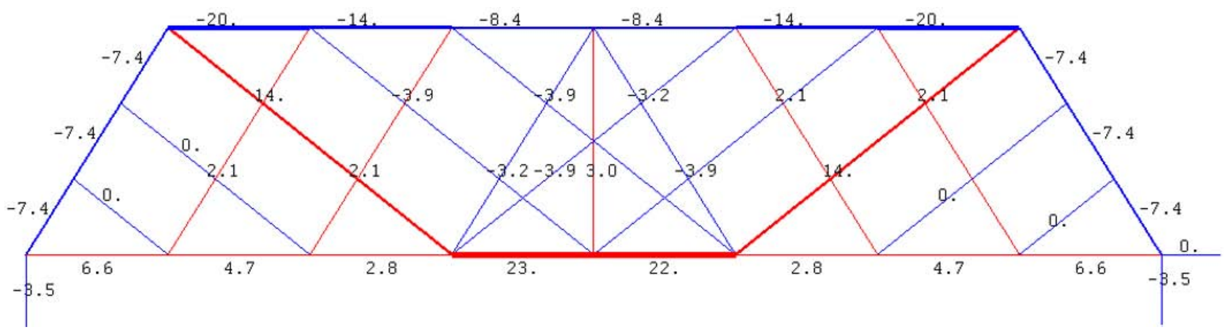

Fig. 4. The distribution of forces S_k/P, $k = 1, \ldots, m$, $n = 3$, $h = 2$ m

Conclusions

The proposed scheme of a statically determinate truss allows an analytic solution of the deflection problem for an arbitrary number of panels. The solution obtained has a simple polynomial form. A linear asymptotic solution is found and a non-monotonous change in the relative deflection is observed as a function of the number of panels. The deflection formula can be used in design practice to assess the deformability of a truss and, as a test for numerical calculations and in solving problems of optimization of structures [10–12].

References

1. *Kachurin, V. K.* About the deflection of bridge trusses / V. K. Kachurin, Sat 17, NTK NKPS. – 1928. (rus)

2. *Ignatiev, V. A.* Calculation of regular core systems / V. A. Ignatiev. – Saratov : Saratov Higher Military Chemical Military School, 1973. – 433 p. (rus)

3. *Domanov, E. V.* The dependence of the deflection of the cantilever truss on the number of panels obtained in the system Maple / E. V. Domanov // Structural mechanics and structures. – 2018. – No. 2. 17. – P. 80–86.

4. *Egorov, S. S.* The inductive method of solving the problem of deflection of the symmetric core structures of complex shape in the system Maple for arbitrary number of panels / S. S. Egorov // Sciense Almanac. – 2017. – 3-3 (29). – P. 254–257. – `doi:10.17117/na.2017.03.03.254`

5. *Kirsanov, M. N.* Analytical calculation of the deflection of the lattice truss / M. N. Kirsanov, D. V. Tinkov // MATEC Web of Conferences. – 2018. – V. 193. – 03015. – `doi.org/10.1051/matecconf/201819303015`

6. *Tinkov, D. V.* Design Optimization of Truss Bridge Structures of Composite Materials / D. V. Tinkov, A. A. Safonov // Journal of Machinery Manufacture and Reliability. – 2017. – V. 46, No 1. – P. 46–52. – `doi:10.3103/S1052618817010149`

7. *Kirsanov, M. N.* Deformations of the periodic truss with diagonal lattice / M. N. Kirsanov, N. V. Zaborskaya // Magazine of Civil Engineering. – 2017. – No. 3. – P. 61–67. – `doi:10.18720/MCE.71.7`

8. *Kirsanov, M. N.* The exact formulas for calculating deflection and forces in the rods of the 'Molodechno' truss with an arbitrary number of panels / M. N. Kirsanov // Magazine of Civil Engineering. – 2016. – No. 1. – P. 33–41. – `doi:10.5862/MCE.61.4`

9. *Kirsanov, M. N.* The deflection of spatial coatings with periodic structure/ M. N. Kirsanov // Magazine of Civil Engineering. – 2017. – No 8. – P. 58–66. – `doi:10.18720/MCE.76.6`

10. *Kaveh, A.* Colliding Bodies Optimization for Size and Topology Optimization of Truss Structures / A. Kaveh, V. R. Mahdavi // Structural Engineering and Mechanics. – 2015. – V. 53 (5). – P. 847–865. – `doi:10.12989/sem.2015.53.5.847`

11. *Kaveh, A.* Sizing, Geometry and Topology Optimization of Trusses Using Force Method and Supervised Charged System Search / A. Kaveh, B. Ahmadi // Structural Engineering and Mechanics. – 2014. – V. 50 (3). – P. 365–382. – `doi:10.12989/sem.2014.50.3.365`

12. *Li, J.-P.* Truss Topology Optimization Using an Improved Species-Conserving Genetic Algorithm / J.-P. Li //Engineering Optimization. – 2015. – V. 47(1). – P. 107–128. – `doi:10.1080/0305215X.2013.875165`