

Formulas for calculating deformations of power line supports

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Abstract. In this work, we investigate the static deformations of the spatial model of a statically determined truss of a power line support. The tetrahedral truss has a pyramidal extension at the base and a cross-shaped lattice. Brackets for attaching the supporting cables are located at the top of the truss. A spherical support hinge, a cylindrical one, and two vertical posts are located at the four corners of the structure base. We consider two types of loads: wind, and force. Horizontal forces applied to the nodes of one face model the wind load. The horizontal force is applied to the top of the structure. We aim to derive formulas for the dependence of the deflections of the truss on the number of its panels. We use the Maxwell-Mohr formula to determine the deflection. We find the efforts in the structural elements and the reactions of the supports from the general system of linear equations of equilibrium of all nodes of the truss. A series of solutions for trusses with different numbers of panels are summarized by the induction method in the Maple computer mathematics system. The sought formulas for the dependence of the vertical deflection of the console and the displacement of the top of the mast on the number of panels were obtained in the form of polynomials in the number of panels of degree not higher than the fourth. Some asymptotics of solutions is found in the work.

1. Introduction

Typically, trusses are used in the construction of transmission towers. Truss engineering calculations are usually performed numerically in various specialized or custom engineering packages. The core of these programs is based on the finite element method [1–6]. An analytical approach can be used to evaluate such solutions and to pre-calculate the designed structures. In this case, the value of the analytical solution is the greater, the more independent parameters were taken into account in the solution (dimensions, material constants, numerical value of the load). For regular structures that have some kind of periodic structure, the order of regularity can also be taken into account, for example, the number of panels. This allows the engineer to select the optimal variant of the designed structure in an analytical form. One of the methods for solving this problem is the induction method [7]. A number of solutions for the statics of flat arches were obtained by this method [8,9]. Researchers are aware of some solutions for planar lattice trusses [10–12] and space trusses [13]. In an analytical form, using the Dunkerley method [14], the lower boundaries of the first natural frequencies of vibrations of regular hinged-bar structures were obtained [15,16]. Some questions of the existence and calculation of statically definable plane and spatial regular trusses were studied in [17–19].

2. Solution method. Truss scheme

The support structure consists of two parts and has a cruciform lattice (figure 1, 2). In the lower pyramidal part of height h , the slope of the faces is determined by the ratio of the horizontal dimensions $a = b + 2c$, c , and the vertical dimension h . The upper part with parallel vertical ribs contains $n-1$ identical belts and belts with a height h_1 for attaching consoles of length r .

Each console consists of six rods. There are a total of members in the truss $m = 12n + 39$, including seven members that simulate the supports. The supports used a spherical hinge A , a cylindrical hinge B , and two vertical posts C and D . We are considering two loading options. The first option is a horizontal force applied to the top of the structure, the second is a lateral horizontal force evenly distributed over the nodes of one face of the truss, simulating a wind load. We set the task to determine in an analytical form the deflection of the cantilever and the horizontal displacement of the top.

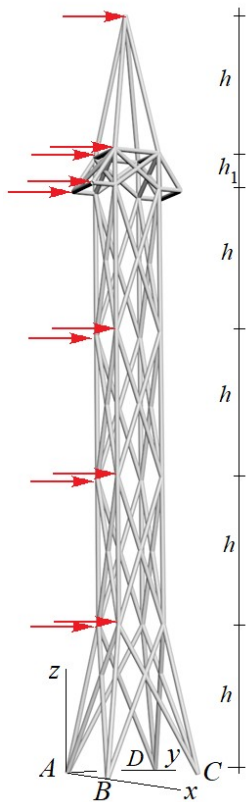


Figure 1. Wind load, truss $n = 4$

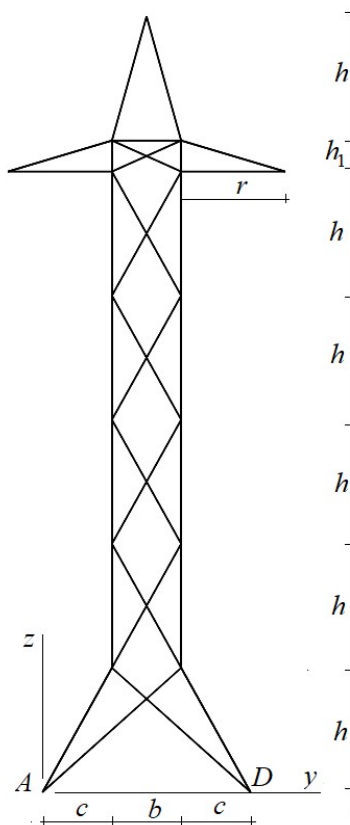


Figure 2. Dimensions, truss $n = 5$

We use a program written in the language of computer mathematics Maple to calculate the forces in the rods [7]. Nodes and rods are numbered. The cycles specify the coordinates of the nodes and the order of connecting the bars in the hinges. To do this, we use the method used for encoding graphs in discrete mathematics. Each bar is associated with a list of node numbers at its ends. We use this data to calculate the direction cosines of the forces connected at the nodes. The matrix \mathbf{G} of the system of equations for the equilibrium of nodes consists of these quantities. The loads on the nodes are entered into the right side of the matrix equation $\mathbf{GS} = \mathbf{T}$. Here \mathbf{S} is the vector of values of all forces in the rods, including seven reactions of the supports. In the matrix, each node has three rows. The system of equations is solved by Maple methods in symbolic form. The Maxwell – Mohr formula is used to determine the displacement of the nodes of the truss

$$\Delta = \sum_{\alpha=1}^{m-7} S_{\alpha}^P S_{\alpha}^1 l_{\alpha} / (EF). \quad (1)$$

The standard notation is used here: S_{α}^1 — the force in the element numbered α from the action of a unit force applied to the node in the direction of the desired displacement, S_{α}^P — the force from the action of a given load, E — modulus of elasticity of the rod material, F — cross-sectional area of bars, l_{α} — rod lengths. It is assumed here that the stiffnesses of the rods are the same. The summation is carried out only over the deformable truss rods, therefore, the efforts of the seven support rods are not included in the sum.

3. Displacement of a vertex under the action of a horizontal force

We will consider the horizontal force P as the load. This force is applied to the top of the structure in the direction of the y -axis. There will be only one nonzero element in the load vector on the right side of the system of equations for the equilibrium of nodes

$$T_{3i-1} = -P, \quad i = 4(n+1) + 5. \quad (2)$$

A preliminary calculation of trusses with a different number of panels n shows that the solution for the deflection calculated according to formula (1) has the same form:

$$\Delta = P(C_1 a^3 + C_2 g^3 + C_3 f^3 + C_4 h^3 + C_5 q^3 + C_6 e^3) / (EFa^2). \quad (3)$$

We have introduced the following designations for the dimensions of the braces

$$r = \sqrt{50a^2 + 64h^2}, \quad f = \sqrt{2a^2 + 16h^2}, \quad g = \sqrt{10a^2 + 16h^2}, \\ p = \sqrt{2a^2 + 64h^2}, \quad q = \sqrt{4a^2 + h^2}, \quad e = \sqrt{a^2 + 4h^2}.$$

The coefficients in (3) C_1, \dots, C_7 depend only on the number of panels in the truss n . These dependencies are determined by the induction method. We have the following sequence of solutions:

$$\Delta_1 = P(3104a^3 + 552f^3 + 73g^3 + 1096h^3 + 552q^3) / (2048EFa^2), \\ \Delta_2 = P(3104a^3 + 9024e^3 + 1128f^3 + 129g^3 + 23240h^3 + 1128q^3) / (2048EFa^2), \\ \Delta_3 = P(9760a^3 + 31360e^3 + 1960f^3 + 217g^3 + 272584h^3 + 1960q^3) / (2048EFa^2), \\ \Delta_4 = P(9760a^3 + 73152e^3 + 3048f^3 + 337g^3 + 578376h^3 + 3048q^3) / (2048EFa^2), \dots$$

We wrote out the first four results, but to identify the regularity of the formation of coefficients, we needed to solve the problem for ten trusses with a sequentially increasing number of panels. We will notice that the speed of symbolic transformations in the Maple system is significantly higher than the speed of numerical calculation. The following dependencies were found by the Maple system operators from the *genfunc* specialized package:

$$C_1 = (16n^2 + 8(3 - 2(-1)^n)n - 12(-1)^n + 29) / 64, \\ C_2 = (16n^2 + 8n + 49) / 2048, \\ C_3 = C_5 = (16n^2 + 24n + 29) / 256, \\ C_4 = (4096n^3 - 720n^2 - 8(426(-1)^n + 359)n - 2556(-1)^n - 6057) / 768, \\ C_6 = (n-1)(16n^2 + 24n + 29) / 32.$$

We have found and solved recurrent equations to obtain these functions. These functions satisfy recurrent equations. The Maple *rsolve* operator gives solutions based on the initial values of the coefficients.

Similarly, in the case of a wind load acting along the y -axis on all nodes of one face of the truss (Fig. 2), we have the coefficients:

$$\begin{aligned}
C_1 &= (16n^3 + 4(15 - 2(-1)^n)n^2 + 8(9 - 4(-1)^n)n + 63) / 64, \\
C_2 &= (16n^3 + 36n^2 + 84n + 139) / 2048, \\
C_3 &= (16n^3 + 60n^2 + 48n + 31) / 256, \\
C_4 &= (1280n^4 + 2576n^3 - 4(142(-1)^n + 657)n^2 - \\
&\quad - 8(314(-1)^n + 231)n - 500(-1)^n - 2809) / 256, \\
C_5 &= (16n^3 + 60n^2 + 72n - 8(-1)^n + 71) / 256, \\
C_6 &= (16n^4 + 44n^3 + 16n^2 - 5n + 4(-1)^n - 67) / 32.
\end{aligned} \tag{4}$$

Here, the lengths of the sequences of solutions required to find their common terms were greater than for the case of one force applied to the vertex.

In figure 3 ($h_1 = h$, $h = H / (n + 3.5)$) the curves of dependence (3) with coefficients (4) for dimensionless relative deflection $\Delta' = EF\Delta / (P_0H)$ were presented. Here is the notation for the total force per mast: $P_0 = (2n + 3)P$.

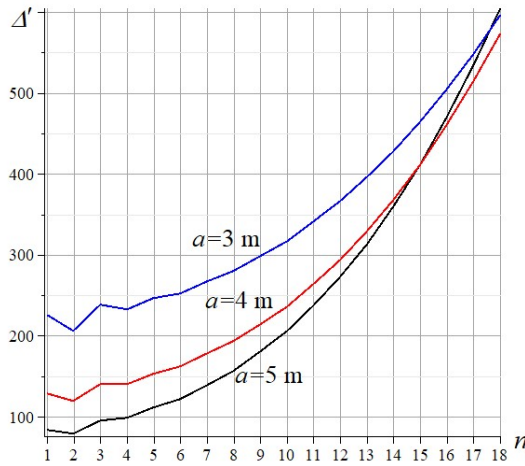


Figure 3. Dependence of the displacement of the top on the number of panels at a uniform lateral load, $H = 20\text{m}$

We can reveal the cubic character of the deflection growth $\lim_{n \rightarrow \infty} \Delta' / n^3 = a / (4H)$ by Maple methods. The uneven growth of the deflection with an increase in the number of panels is explained by the presence of terms with a multiplier $(-1)^n$ in the solution. The intersections of the curves plotted for different base sizes show the ambiguity in the choice of the size a at a constant height and load. The offset for different values of a and the same number n of panels may turn out to be the same. If we do not consider here the problem of overturning the structure with an insufficiently wide base of the truss, then the solution with a lower a will be optimal from the point of view of saving material.

4. Console deflection due to lateral loads

The trusses are tilted under the action of a horizontal load. We calculate the vertical displacement of the end of the leeward console using the same formula (1). The solution to the problem of displacement of the console is similar to the solution (3), the solution differs in the denominator, and has the form:

$$\Delta = P(C_1a^3 + C_2g^3 + C_3f^3 + C_4h^3 + C_5q^3 + C_6e^3) / (EFah). \tag{5}$$

The coefficients in this expression are obtained by the induction method and have the form

$$C_1 = (8n^2 + 4(9 - 8(-1)^n)n - 57(-1)^n + 15) / 64,$$

$$C_2 = (2n + 7)(2n + 1) / 1024,$$

$$C_3 = (2n + 7)(2n + 1) / 128,$$

$$C_4 = (512n^3 + 2(963 - 87(-1)^n)n^2 - 26(27(-1)^n + 70)n - 522(-1)^n - 1983) / 192,$$

$$C_5 = (8n^2 + 4(9 - 4(-1)^n)n - 31(-1)^n + 39) / 256,$$

$$C_6 = (16n^3 + 56n^2 - 26n + 3(-1)^n - 43) / 64.$$

In figure 4, we show the change in the relative displacement versus the number of panels for different base sizes a . As in the previous problem, the growth of the quantity is cubic. This shows the following limit: $\lim_{n \rightarrow \infty} \Delta' / n^3 = 3a^2 / (16H^2)$, $h = H / (n + 3.5)$. A characteristic feature of the obtained solution is its non-monotonicity. Changing the number of panels at a total constant height by just one can see noticeably change the deflection. As in the previous solution, the curves have points of intersection with each other.

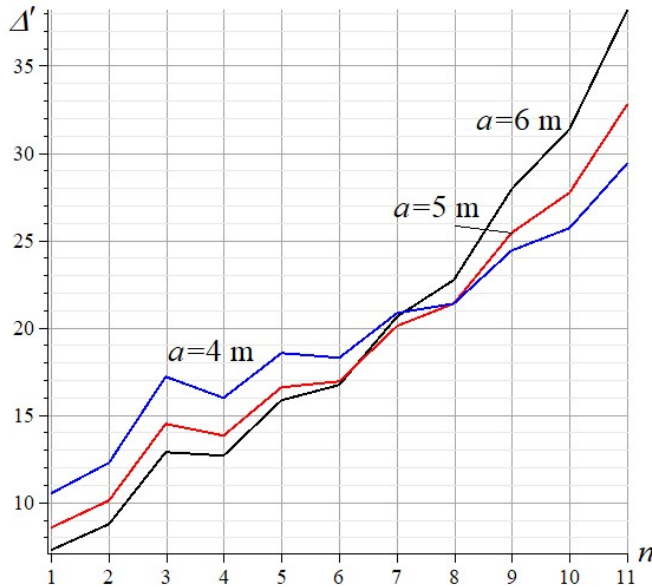


Figure 4. Dependence of the cantilever deflection on the number of panels under the action of a wind load, $H = 20\text{m}$

Analytical solutions for the calculation of building structures can be used in solving optimization problems [5,20–22]. The presence of a parameter in the solution that is responsible for the number of panels significantly expands the class of structures under study and significantly accelerates the search for the optimal solution.

5. Conclusion

The main results of the work are as follows.

1. We have considered and studied a mathematical model of a statically determinate spatial truss of a transmission line support.
2. Formulas for calculating the deflection of the console and the displacement of the top of the structure under the influence of distributed wind and concentrated loads are derived.
3. We have found cubic asymptotics of solutions.

6. Acknowledgments

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References

- [1] Itam Z, Beddu S, Mohd Kamal N L and Bamashmos K H 2016 Finite Element Analysis of the Maximum Stress at the Joints of the Transmission Tower *IOP Conference Series: Earth and Environmental Science* vol 32 (Institute of Physics Publishing)
- [2] Shu Q, Huang Z, Yuan G, Ma W, Ye S and Zhou J 2018 Impact of wind loads on the resistance capacity of the transmission tower subjected to ground surface deformations *Thin-Walled Struct.* **131** 619–30
- [3] Zeynalian M, Bolkhari S and Rafeei P 2018 Structural performance of cold-formed steel trusses used in electric power substations *J. Constr. Steel Res.* **147** 53–61
- [4] Vatin N I and Sinelnikov A S 2012 Footway bridges: cold formed steel cross-section *Constr. Unique Build. Struct.* **3** 39–51
- [5] Tyukalov Y 2020 Optimal Shape of Arch Concrete Block Bridge *Constr. Unique Build. Struct.* **93** 9307
- [6] Degtyareva N 2021 Finite Element Analysis of Cold-Formed Steel Channels with Solid and Slotted Webs Subjected to Web Crippling under Interior Two Flange Load Case *Constr. Unique Build. Struct.* **95** 9502–9502
- [7] Buka-Vaivade K, Kirsanov M N and Serdjuks D O 2020 Calculation of deformations of a cantilever-frame planar truss model with an arbitrary number of panels *Vestn. MGSU* **4** 510–7
- [8] Kirsanov M 2020 *Trussed Frames and Arches: Schemes and Formulas* (Newcastle upon Tyne, GB: Cambridge Scholars Publishing Lady Stephenson Library)
- [9] Kirsanov M 2019 *Planar Trusses: Schemes and Formulas* (Newcastle upon Tyne, GB: Cambridge Scholars Publishing Lady Stephenson Library)
- [10] Ovsyannikova V M 2020 Dependence of the deflection of a planar external statically undeterminable truss on the number of panels *Struct. Mech. Struct.* **27** 16–25
- [11] Petrichenko E A 2020 Lower bound of the natural oscillation frequency of the Fink truss *Struct. Mech. Struct.* **26** 21–9
- [12] Vorobyev O 2020 About methods of obtaining analytical solution for eigenfrequencies problem of trusses *Struct. Mech. Struct.* **1** 25–38
- [13] Kirsanov M N 2021 Deformations of the Rod Pyramid: An Analytical Solution *Constr. Unique Build. Struct.* **95** 9501–9501
- [14] Low K H 2000 Modified Dunkerley formula for eigenfrequencies of beams carrying concentrated masses *Int. J. Mech. Sci.* **42** 1287–305
- [15] Vorobev O V 2020 Bilateral Analytical Estimation of the First Frequency of a Plane Truss *Constr. Unique Build. Struct.* **92** 9204–9204
- [16] Kirsanov, M. N., Vorobyev O V 2021 Calculating of a spatial cantilever truss natural vibration frequency with an arbitrary number of panels: analytical solution *Constr. Unique Build. Struct.* **94** 9402
- [17] Hutchinson R G and Fleck N A 2006 The structural performance of the periodic truss *J. Mech. Phys. Solids* **54** 756–82
- [18] Hutchinson R G and Fleck N A 2005 Microarchitected cellular solids - The hunt for statically determinate periodic trusses *ZAMM Zeitschrift fur Angew. Math. und Mech.* **85** 607–17
- [19] Zok F W, Latture R M and Begley M R 2016 Periodic truss structures *J. Mech. Phys. Solids* **96** 184–203
- [20] Martins A M B, Simões L M C and Negrão J H J O 2021 Optimization of extradosed concrete bridges subjected to seismic action *Comput. Struct.* **245**

- [21] Gomes H M 2011 Truss optimization with dynamic constraints using a particle swarm algorithm *Expert Syst. Appl.* **38** 957–68
- [22] Khatibinia, M., Naserlavi S S 2014 Truss optimization on shape and sizing with frequency constraints based on orthogonal multi-gravitational search algorithm *J. Sound Vib.* **333** 6349–69