Analytical Dependence of Deflection of the Lattice Truss on the Number of Panels

Mikhail Kirsanov, Dmitrijs Serdjuks and Karina Buka-Vaivade

Abstract The case of load in the middle of the span is considered. An algorithm is given for deriving the analytical dependence of the deflection of a truss on the number of panels. It is shown that for a certain combination of panel numbers over the height of the truss and along the length of the span, the determinant of the system of equations of node equilibrium turns to zero that corresponds to the kinematic variability of the structure. It is noted that numerical calculation can obscure this feature due to inaccuracy of calculations. To obtain an exact formula, the computer mathematics system Maple is used. The coefficients of the final formula are obtained from the solution of linear homogeneous recurrent equations found from the analysis of the sequence of solutions for trusses with different number of panels. An example of a scheme of possible velocities of nodes of a variable truss is given. The formula for the deflection of the truss for the number of panels for which the truss is unchanged is derived by induction.

Keywords Truss · Frame · Deflection · Kinematic variability · Induction · Maple

1 Introduction

The problem of searching for schemes of statically definable trusses was first raised by Hutchinson and Fleck [1, 2]. Regular or periodic trusses, which have repeating elements in the structure, allow inductive analysis of analytical dependences of deflections and forces in the rods of trusses. Some constructions turn into instantly changing mechanisms at certain numbers of panels [3, 4]. This makes us use caution when designing designs known formulas and numerical solutions obtained for a single number of panels for truss with a different number of panels. A truss working in the first case, in the second, with a changed number of panels becomes a mechanism

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that is not suitable as a load-bearing structure. Most analytical solutions for deflecting inverted trusses are controlled by one natural parameter—the number of panels in the span. Solutions with two parameters are less common. Usually, these are frames and arches, where the second parameter can be the number of panels along the height of the truss.

The analytical solution of the problem of deformation of the truss completely does not replace the numerical one. It can be successfully used, for example, for a simple assessment of the deformations of existing structures [5], in optimization problems [6–9] and in problems on suspended structures of regular type [10], to control numerical solutions made in special packages based on the finite element method [5, 11–14]. An overview of some papers using the induction method for deriving analytical solutions for planar trusses is contained in [15].

The objective of investigation is to derive the analytical dependence of the deflection of the lattice truss on the number of panels. Similar solutions by induction were obtained for plane trusses [16–22] and for spatial ones [23].

2 Methods

2.1 Object of Investigation

A planar statically determinate truss has a span of length $L = 2(n + 3)a$ and height $(m + 2.5)h$ (Fig. 1). The lattice consists of parallel braces with a tangent of slope h/a. The rods of the lower and upper belts have a length of $2a$, and the lateral posts have a height h. In the truss, there are rods. In the middle of the span, the truss is loaded with the force P.

To derive the analytical dependence of the deflection of the structure on the number of panels, we need to perform induction on two parameters m and n. It is proposed to obtain a series of solutions for trusses with different number of panels, to reveal the sequence of the corresponding coefficients in the expressions for the deflection, and then to find the common terms of these sequences. To determine the deflection, the Maxwell-Mohr's formula is used:

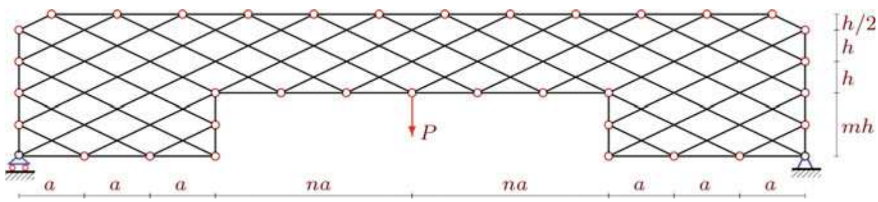

Fig. 1 Truss at $n = 3$, $m = 2$

$$\Delta = P\sum_{i=1}^{M-3} S_i^2 l_i/(\mathrm{EF}) \qquad (1)$$

where l_i—the length of the rods, S_i—the forces from the unit force applied to the knot of the lower belt in the middle of the span, EF—the rigidity of the rods.

It is assumed that the rods of the truss have the same rigidity, and the supports are modelled by rigid, non-deformable rods. The values of the forces entering into the sum of the Maxwell-Mohr's formula are calculated in a symbolic form in the computer mathematics system Maple on the basis of the program [3].

2.2 Calculation

The method of induction involves the calculation of individual trusses with a consecutively increasing number of panels. Data on the truss with the number of panels $2n$ in the crossbar and m on the height of the truss are entered in the program. The rods and hinges are numbered (Fig. 2).

The coordinates of the hinges on the sides of the truss ($i = 1, 2, \ldots, m + 3$) are $x_i = 0$, $y_i = (i - 1)h$ and $x_{i+2n+m+9} = 2(n + 3)a$, $y_{i+2n+m+9} = (m + 3 - i)h$. Coordinates of nodes of the upper belt are $x_{i+m+3} = (i - 0.5)a$, $y_{i+m+3} = (m + 2.5)h$, $i = 1, \ldots, 2n + 6$.

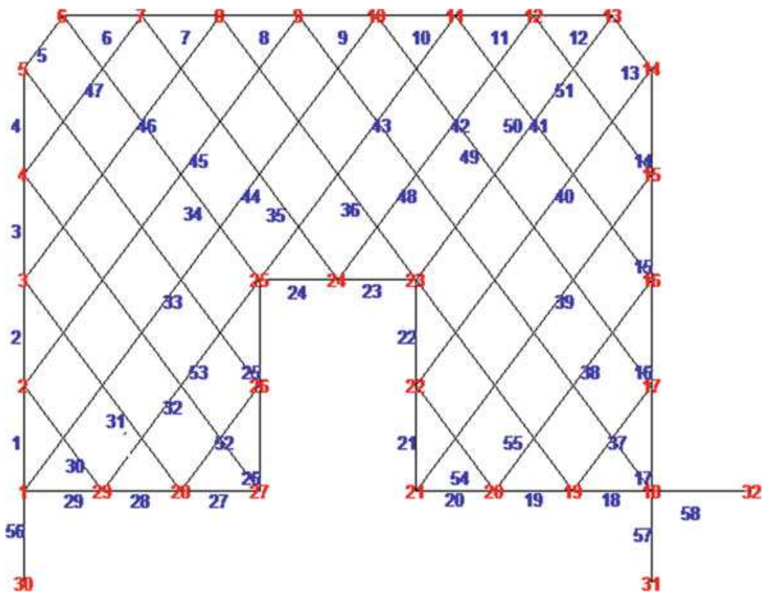

Fig. 2 Numbering of the nodes and rods of the truss at $n = 1$, $m = 2$

The order of joining the rods and hinges is determined by special vectors $\{\mathbf{N_i}\}$ containing in their coordinates the numbers of the hinges at their ends in the same way as in the discrete mathematics the graph is given. Thus, lateral rods and rods of the upper belt are encoded by vectors $\{\mathbf{N_i}\}= \{\mathbf{i}, \mathbf{i}+\mathbf{1}\}$, $i = 1, \ldots, 4(n + m + 4)$.

The system of equations for the equilibrium of nodes is written in a matrix form $G\{\mathbf{S}\} = \{\mathbf{B}\}$, where G is the matrix of cosine-directed forces in the rods, $\{\mathbf{S}\}$ is the force vector, and $\{\mathbf{B}\}$ is the load vector. In the odd rows of the matrix, the projections of the vectors $\{\mathbf{N_i}\}$ onto the horizontal axis are introduced, and the even ones onto the vertical axis. The matrix G is filled in the same way:

$$G_{t,i} = -(x_{N_{i,2}} - x_{N_{i,1}})/l_i, \quad t = 2N_{i,2} - 2 + j, \quad t < M, \quad j = 1, 2;$$
$$G_{t,i} = (x_{N_{i,2}} - x_{N_{i,1}})/l_i, \quad t = 2N_{i,1} - 2 + j, \quad t < M, \quad j = 1, 2.$$

The solution of the system of linear equations can be obtained by special operators of the Maple system from the LinearAlgebra package, but the inverse matrix method $\{\mathbf{S}\} = \{\mathbf{B}\}/G$.

2.3 Numerical Calculations and Kinematic Changeability

The first debugging calculations showed that the results for some values of n depend significantly on the accuracy of the calculations. In the Maple system, the precision determines the Digits parameter, and the default is 10. The dependence of the dimensionless deflection $\Delta' = \Delta \mathrm{EF}/(\mathrm{PL})$ at $m = 1$, $a = h = 2$ m on the number of panels in the crossbar was nonmonotonic. Moreover, the repeated calculation gave a different result at some points (Fig. 3).

Calculation in a symbolic form showed that for these values the determinant of the system turns to zero, which indicates a kinematic changeability of the truss. The corresponding scheme of the possible speeds of the nodes of the truss is also found (Fig. 4). Most of the nodes, including the support ones, remain stationary, and the rest are displaced. For example, rods 2–7, 2–33 acquire rotational motion, and rods 9–33, 9–26 acquire instantaneous translational motion.

Rectangular lattice truss that marked with a sufficient sign of the kinematic variability of the structure has the presence of a closed chain of rods that does not pass through the corner points of the structure [24]. In the investigated truss, this is confirmed by the chain 2-7-28-12-21-19-14-26-9-33-2. Trial calculations (numerical and analytical) have shown that in the general case when

$$m = 3j - 1, \quad j = 1, 2, \ldots, n = [10k - 5 - 3(-1)^k]/4, \quad k = 1, 2, \ldots \tag{2}$$

the determinant of the system of equilibrium equations is nonzero. At these values will be carried out induction to obtain the general formula.

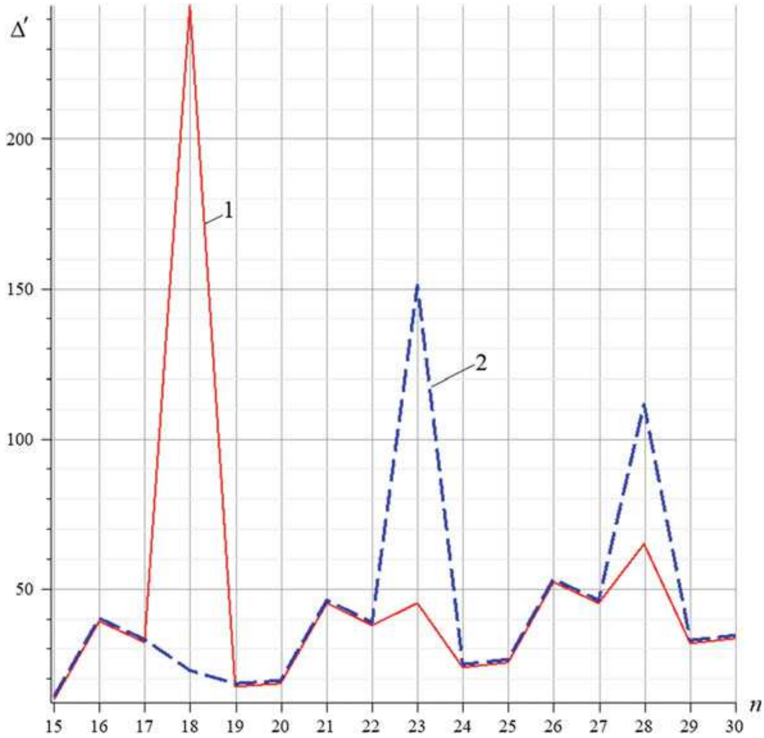

Fig. 3 Dependence of the deflection on the number of panels n, obtained numerically. 1—digits = 6; 2—digits = 7. For $n = 18, 23, 28$, the determinant of the system of equations becomes zero. Numerical calculations at these values give an erroneous result

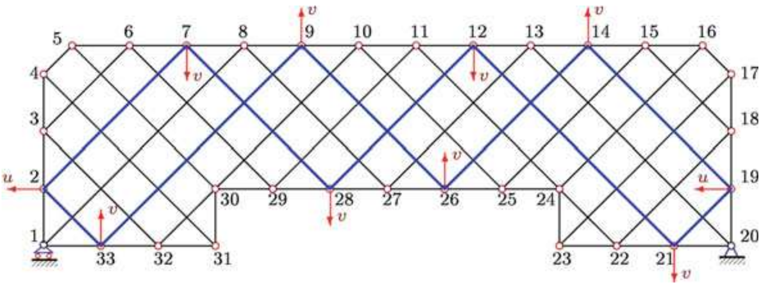

Fig. 4 Scheme of possible velocities of truss nodes, $n = 3$, $m = 1$

2.4 Induction

Consider the class of trusses (2) for $k = j$. The considered truss belongs to regular, for which the form of the solution does not depend on the number of panels. In this case, the first calculations according to the formula (1) give the following form for the deflection:

$$\Delta = P(C_1 a^3 + C_2 c^3 + C_3 h^3)/(4h^2 \mathrm{EF}) \tag{3}$$

where $c = \sqrt{a^2 + h^2}$. It only remains to obtain the dependence of the coefficients in this expression on the number k, related by the expression (2) with the number of panels. To obtain a sequence of coefficients $C_{1,i}$, $i = 1, 2, \ldots$ of sufficient length at which the general term formula can be found, it was necessary to calculate 14 trusses. The sequence 420, 380, 962, 986, 1856, 2072, 3262, 3798, 5340, 6324, 825, … was obtained. The rgf_findrecur operator of the Maple system yields a linear homogeneous recurrence equation that is satisfied by the terms of the sequence

$$C_{1,k} = C_{1,k-1} + 3C_{1,k-2} - 3C_{1,k-3} - 3C_{1,k-4} + 3C_{1,k-5} + C_{1,k-6} - C_{1,k-7}. \tag{4}$$

The solution of this equation is found by the operator rsolve:

$$C_1 = (20k^3 + 6(7 - 3(-1)^k)k^2 + 2(320 + 141(-1)^k)k + 273 + 147(-1)^k)/6. \tag{5}$$

Similarly, from simpler equations, we find expressions for two other coefficients:

$$\begin{aligned} C_2 &= (6(80(-1)^k + 143)k - 15(-1)^k - 21)/2, \\ C_3 &= 2((24(-1)^k + 49)k + 13(-1)^k + 14). \end{aligned} \tag{6}$$

A different class of solutions is also found that corresponds to combinations of panel numbers, in which the determinant of the system of equilibrium equations does not vanish:

$$\begin{aligned} &m = 3k + 1, \\ &n = (10k - 1 - \cos 2\varphi + 2\cos\varphi - 2\sin\varphi)/8, \quad k = 1, 2, 3, \ldots \\ &\varphi = \pi k/2. \end{aligned}$$

The coefficients in (3) in this case have the form:

$$\begin{aligned} C_1 = {} & (10k^3 + (69 - 3\cos 2\varphi + 6\cos\varphi - 6\sin\varphi)k^2 + \\ & + (63\cos 2\varphi - 588\cos\varphi + 432\sin\varphi + 833)k + \\ & + 78\cos 2\varphi - 804\cos\varphi + 522\sin\varphi + 1158)/24, \\ C_2 = {} & ((602 - 40\cos 2\varphi - 368\cos\varphi + 448\sin\varphi)k - \\ & - 5\cos 2\varphi - 246\cos\varphi + 246\sin\varphi + 419)/4, \\ C_3 = {} & 2((8\cos 2\varphi - 32\cos\varphi + 16\sin\varphi + 33)k + \\ & + \cos 2\varphi - 8\cos\varphi + 8\sin\varphi + 16). \end{aligned}$$

The recurrence equations, from the solution of which these expressions were obtained, turned out to be more complicated. Thus, to derive the coefficient C_1, it was necessary to analyse 26 solutions and solve equation:

$$C_{1,k} = C_{1,k-1} + 3C_{1,k-4} - 3C_{1,k-5} - 3C_{1,k-8} + 3C_{1,k-9} + C_{1,k-12} - C_{1,k-13}. \quad (7)$$

Note that the symbolic transformations in the Maple system are much slower than the numerical ones. If it took a little less than an hour to derive Eq. (4), Eq. (7), directly and sequentially increasing the number of panels in the truss, could not be obtained after several hours of computer operation (i7 processor with good memory). To reduce the time of the transformations, the following method was used. The general form of the coefficients C_2, C_3 turned out to be quite fast, which made it possible to convert the count to numerical mode by specifying the numerical values of the dimensions $a = 1$ and $h = 1$. After that, the coefficient at a^3 can be calculated as an expression $4h^2\Delta\mathrm{EF}/P - C_2c^3 - C_3h^3$. This technique does not just speed up the derivation of the formula, but there is only one way to get the result if you limit research to hours, not days and weeks.

The third and final class of solutions corresponding to admissible combinations of panel numbers has the form:

$$\begin{aligned} &m = 3k, \\ &n = (10k + 1 + \cos 2\varphi - 2\cos\varphi - 2\sin\varphi)/8, \quad k = 1, 2, 3, \ldots \end{aligned}$$

The corresponding coefficients have the form:

$$\begin{aligned} C_1 &= (10k^3 + (75 - 6\cos\varphi + 3\cos 2\varphi - 6\sin\varphi)k^2 + \\ &\quad + (401 - 108\cos\varphi + 15\cos 2\varphi + 48\sin\varphi)k - \\ &\quad - 252\cos\varphi - 6\cos 2\varphi + 138\sin\varphi + 690)/24, \\ C_2 &= (194k + 125 - 10\cos\varphi - 10\sin\varphi + 5\cos 2\varphi)/4, \\ C_3 &= 2((17 - 8\cos\varphi + 8\sin\varphi)k + 4\sin\varphi - 3\cos 2\varphi + 7). \end{aligned}$$

3 Results and Discussion

Let us consider in more detail the solution of (3) with the coefficients (4) and (5). Let the height of the truss be fixed and regardless of the number of panels be $H =$ 20 m. In this case, $h = H/(m + 2.5)$. The horizontal dimension of panel a is also determined by the span length $a = L/(2n + 6)$. Figure 5 shows the curves of the obtained dependence for different spans.

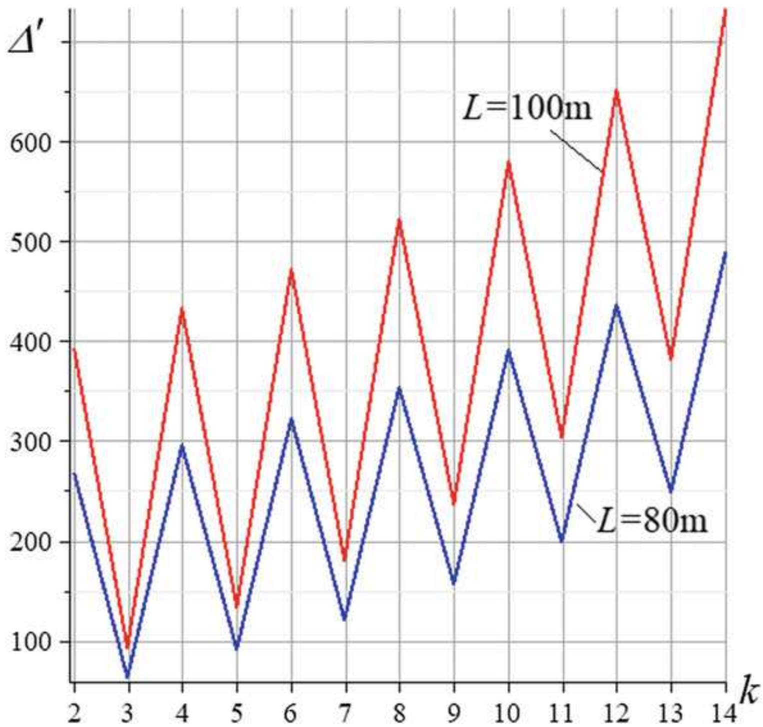

Fig. 5 Dependence of the dimensionless deflection on the number *k*

The big jumps in the solution are characteristic. The segments connecting the individual solution points are given only for clarity. The solution is defined on the set of natural numbers *k*. Moreover, a small change in the number of panels can lead to the fact that the deflection of a structure with a larger span value will be smaller than with a shorter span. Of course, only those values of *k* for which the solution exists are taken. If we construct an analogous curve as a function of *n* and not of *k*, then at many points the solution does not exist because of the kinematic variability of the structure.

On the curves of the dependence of the relative deflection on the height of the truss (Fig. 6), there are minimum points. In this case, the curves constructed for different values of spans (from 40 to 50 m) after these points alternate. For small truss heights (in this example, $h < 2.3 \ldots 2.7$), the greater the deflection, the larger the span. After points of reaching a minimum, the picture changes to the opposite.

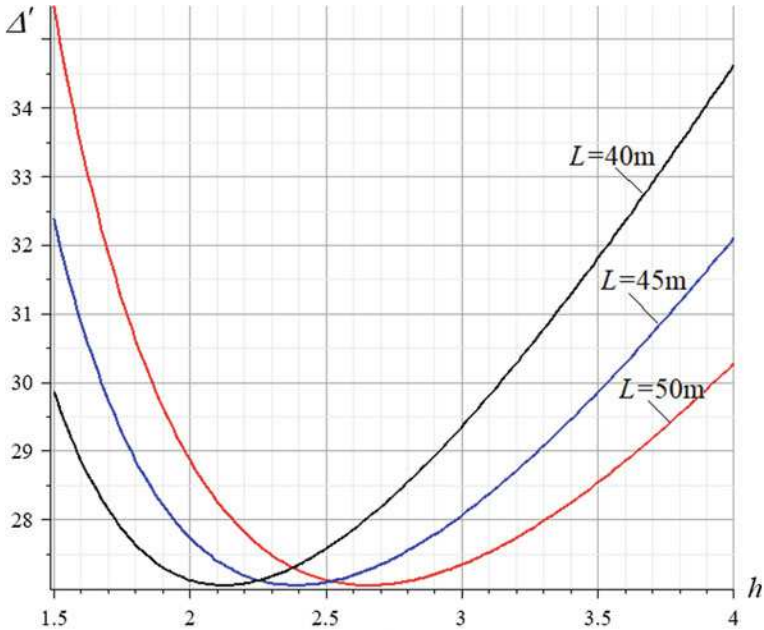

Fig. 6 Dependence of the dimensionless deflection on the height h for $k = 5$

4 Conclusions

A new scheme of a statically determinate planar truss is proposed. The analytical dependence of the deflection on the number of panels is obtained. It is shown that:

1. The truss has a hidden defect, consisting in the fact that with a certain combination of the number of panels along the length and height, the determinant of the system of linear equilibrium equations turns to zero, which indicates the kinematic variability of the truss. The corresponding velocity distribution of nodes is found, which confirms this effect.
2. The nonmonotonic nature of the solution hides the degeneracy of the determinant behind the error in calculations if the solution is performed numerically, what confirms the importance of analytical solutions, applicable to any values of the number of panels without loss of accuracy of the result.
3. In comparison with the known solutions obtained for similar truss by the induction method using the Maple system, the dependences found are much more complicated.
4. The solution was obtained only for one type of loading in the case of one-parameter induction. Theoretically, two-parameter induction (with respect to the independent parameters n and m) for this truss is possible, but in practice the expressions for the deflection become very cumbersome and not very convenient.

References

1. Hutchinson, R.G., Fleck, N.A.: Microarchitectured cellular solids—the hunt for statically determinate periodic trusses. ZAMM Z. Angew. Math. Mech. **85**, 607–617 (2005). https://doi.org/10.1002/zamm.200410208
2. Hutchinson, R.G., Fleck, N.A.: The structural performance of the periodic truss. J. Mech. Phys. Solids **54**, 756–782 (2006). https://doi.org/10.1016/j.jmps.2005.10.008
3. Kirsanov, M.N.: Analysis of the deflection of a strut-type lattice girder truss. Mag. Civ. Eng. **57**(5), 58–65 (2015). https://doi.org/10.5862/MCE.57.5
4. Kirsanov, M.N., Zaborskaya, N.: Deformations of the periodic truss with diagonal lattice. Mag. Civ. Eng. **71**(3), 61–67 (2017). https://doi.org/10.18720/MCE.71.7
5. Kilikevičius, A., Bačinskas, D., Jurevičius, M., Kilikevičienė, K., Fursenko, A., Jakaitis, J., Točka, E.: Field testing and dynamic analysis of old continuous truss steel bridge. Balt. J. Road Bridg. Eng. **13**, 54–66 (2018). https://doi.org/10.3846/bjrbe.2018.394
6. Kaveh, A., Ahmadi, B.: Sizing, geometry and topology optimization of trusses using force method and supervised charged system search. Struct. Eng. Mech. **50**, 365–382 (2014). https://doi.org/10.12989/sem.2014.50.3.365
7. Tinkov, D.V., Safonov, A.A.: Design optimization of truss bridge structures of composite materials. J. Mach. Manuf. Reliab. **46**, 46–52 (2017). https://doi.org/10.3103/S1052618817010149
8. Saknite, T., Serdjuks, D., Goremikins, V., Pakrastins, L., Vatin, N.I.: Fire design of arch-type timber roof. Mag. Civ. Eng. **64**(4), 26–39 (2016). https://doi.org/10.5862/MCE.64.3
9. Goremikins, V., Serdjuks, D., Buka-Vaivade, K., Pakrastins, L.: Choice of rational structural solution for smart innovative suspension structure. Procedia Eng. **172**, 1212–1219 (2017). https://doi.org/10.1088/1757-899X/251/1/012083
10. Krivoshapko, S.: Suspension cable structures and roofs of erections. Constr. Unique Build. Struct. **34**(7), 51–70 (2015) (in Russian)
11. Alekseytsev, A.V., Kurchenko, N.S.: Deformations of steel roof trusses under shock emergency action. Mag. Civ. Eng. **73**(5), 3–13 (2017). https://doi.org/10.18720/MCE.73.1
12. Rybakov, V.A., Al Ali, M., Panteleev, A.P., Fedotova, K.A., Smirnov, A.V.: Bearing capacity of rafter systems made of steel thin-walled structures in attic roofs. Mag. Civ. Eng. **76**(8), 28–39 (2017). https://doi.org/10.18720/MCE.76.3
13. Tusnina, O.A.: Finite element analysis of crane secondary truss. Mag. Civ. Eng. **77**(1), 68–89 (2018). https://doi.org/10.18720/MCE.77.7
14. Indeykin, I.A., Chizhov, S.V., Shestakova, E.B., Antonyuk, A.A., Evtukov, E.S., Kulagin, K.N., Karpov, V.V., Golitsynsky, G.D.: Dynamic stability of the lattice truss of the bridge taking into account local oscillations. Mag. Civ. Eng. **76**(8), 266–278 (2017). https://doi.org/10.18720/MCE.76.23
15. Tinkov, D.V.: Comparative analysis of analytical solutions to the problem of truss structure deflection. Mag. Civ. Eng. **57**(5), 66–73 (2015). https://doi.org/10.5862/MCE.57.6
16. Arutyunyan, V.B.: Calculation of the deflection of a statically indeterminate beam truss. Postulat **6** (2018)
17. Domanov, E.V.: The dependence of the deflection of the cantilever truss on the number of panels obtained in the system Maple. Struct. Mech. Struct. **17**(2), 80–86 (2018)
18. Kitaev, S.S.: Derivation of the formula for the deflection of a cantilevered truss with a rectangular diagonal grid in the computer mathematics system Maple. Postulat **5** (2018)
19. Shipaeva, A.S.: Calculation of the deflection of girder beam loaded on the bottom flange in the system Maple. Sci. Almanac **5–3**(19), 236–239 (2016). https://doi.org/10.17117/na.2016.05.03.236
20. Voropay, R.A.: Analytical calculation of the deflection of a beam truss with parallel belts. Postulat **6** (2018)
21. Rakhmatulina, A.R., Smirnova, A.A.: Analytical calculation and analysis of planar springel truss. Struct. Mech. Struct. **17**(2), 72–79 (2018)

22. Rassokhin, A.S., Ponomarev, A.N., Figovsky, O.L.: The formation of the seabed surface relief near the gravitational object. Mag. Civ. Eng. **79**(3), 132–139 (2018). https://doi.org/10.18720/MCE.79.14
23. Kirsanov, M.N.: Analysis of the buckling of spatial truss with cross lattice. Mag. Civ. Eng. **64**, 52–58 (2016). https://doi.org/10.5862/MCE.64.5
24. Rabinovich, I.M.: Kurs stroitel'noy mekhaniki sterzhnevykh sistem. Stroyizdat, Moscow-Leningrad (1950)