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МАТЕМАТИЧЕСКАЯ МОДЕЛЬ НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ СТЕРЖНЕВОЙ КОНСТРУКЦИИ
(Mathematical model stress-strain state of the rod structure)

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Abstract. A statically determinate flat truss models a set of industrial objects. To derive the dependence of the design deflection on the number of panels in the span, the Maxwell-Mora formula, the Maple computer mathematics system, and the induction method are used. The case of a concentrated load is considered. The formula for the displacement of the support is derived.

Keywords: truss, Maple, deflection, induction, exact solution

Calculation of rod designs with a large number of elements is usually performed in specialized packages based on the finite element method [1]. Without neglecting the role of numerical calculations, we note that recently, with the development of computer mathematics (Maple, Mathematica, MatCad, etc.), a real possibility of analytical calculations with compact resulting formulas for efforts in critical (most compressed or stretched) rods, deflection and displacement of supports. The most effective solutions are for regular periodic structures, for which a natural parameter equal, for example, to the number of periodicity cells can be added to the solution.

Formulation of the problem. The truss in Figure 1 is a model of the roof of the industrial facility.

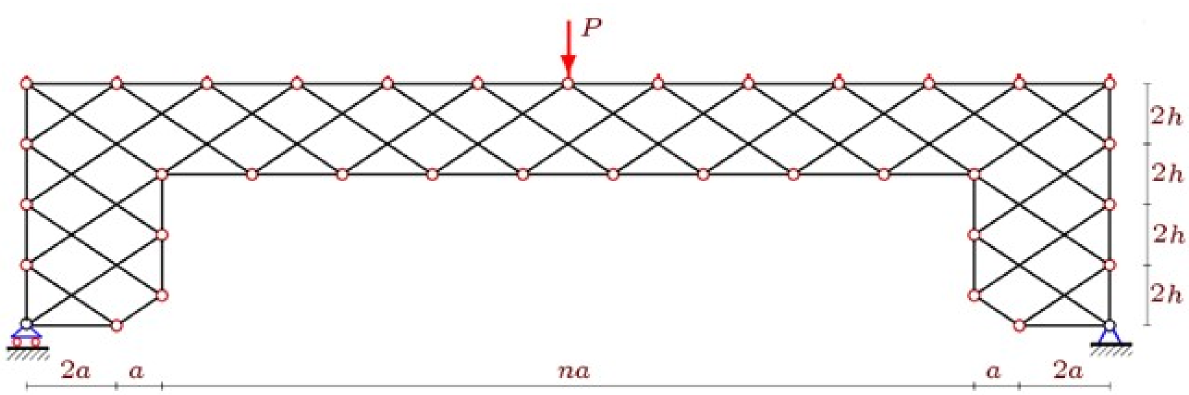

Figure 1 – truss, n=9

The calculation is made on the load evenly distributed over the upper belt. We take an odd number of panels $n = 2k$-1 in the lower belt. The supporting parts of the structure are also farms. We shall derive the formula for the dependence of the deflection, measured from the displacement of the middle node of the upper belt, on the number of panels. In this paper, we take four panels for the height of the structure, in the general case this number may be different. The truss contains $m = 4n + 38$ rods, including three support rods. Efforts in the rods are calculated according to the program [2]. Entering data into the program begins with specifying the coordinates of the hinges (Fig. 2).

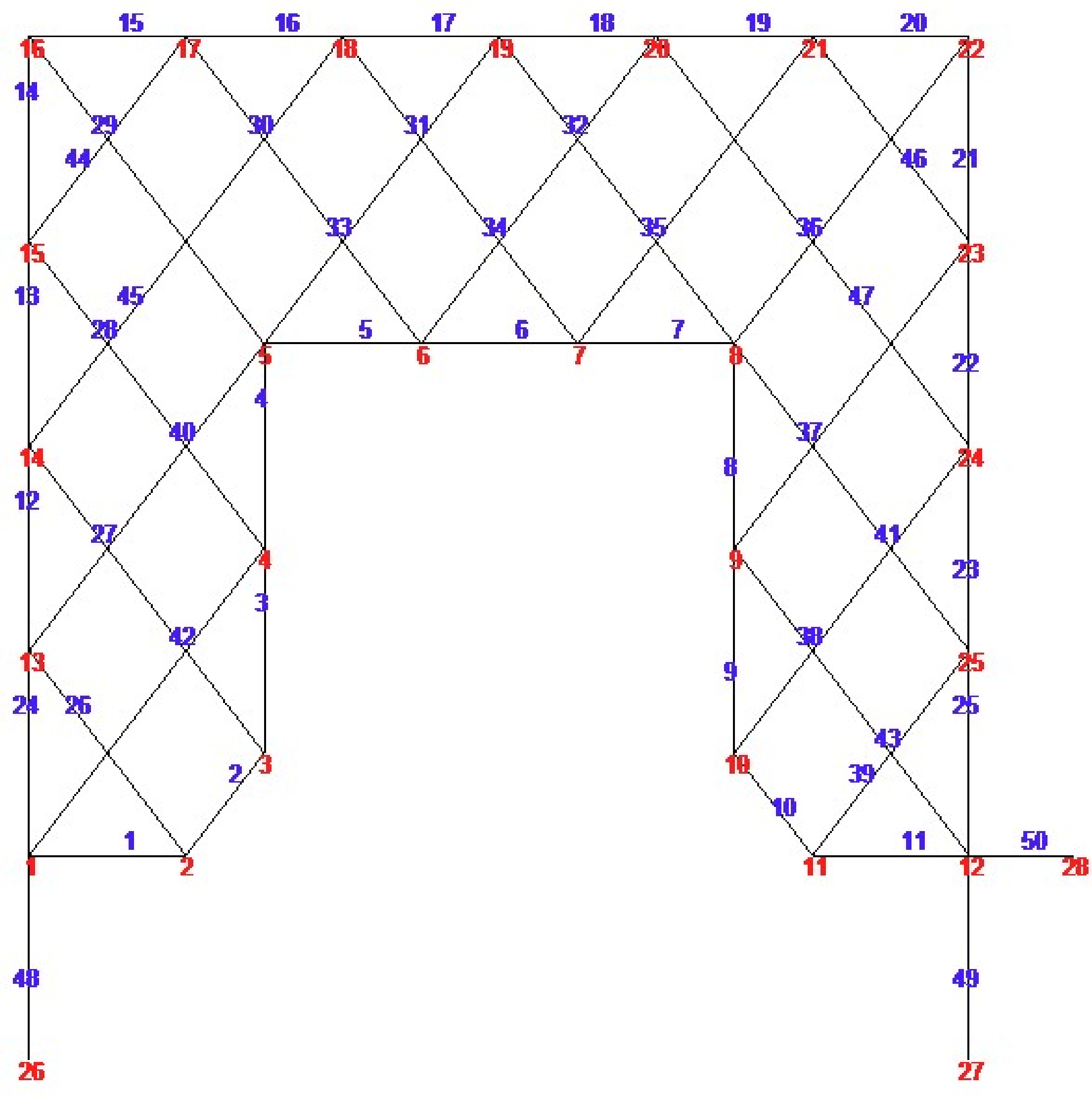

Figure 2 - numbering of nodes $n = 3$, $k = 2$

Here is a fragment of the program in Maple [3]:

```
> x[1]:=0:y[1]:=0:x[2]:=2*a:y[2]:=0:
> for i to 3 do
>  x[i+2]:=3*a;            y[i+2]:=h*(2*i-1);
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```
> x[i+n+4]:=a*(3+2*n);   y[i+n+4]:=h*(7-2*i);
> od:
> x[n+8]:=4*a+2*n*a: y[n+8]:=0:
> x[n+9]:=6*a+2*n*a: y[n+9]:=0:
> for i to n-1 do
> x[i+5]:=2*a*i+3*a; y[i+5]:=5*h; od:
> for i to 4 do
> x[i+n+9]:=0;              y[i+n+9]:=2*h*i;
> x[i+2*n+15]:=2*a*(n+3); y[i+2*n+15]:=10*h-2*h*i;
> od:
> for i to n+2 do
> x[i+n+13]:=2*a*i;   y[i+n+13]:=8*h;
> od:
```

The lattice of the truss is given by the same principle that the structure of graphs in discrete mathematics is the assignment of special vectors containing the numbers of the ends of the rod. So the rods on the inner sides of the supporting elements and the lower belt (Figure 2) are represented by vectors

```
for i to 2*m+n+2 do N[i]:=[i,i+1];od:
```

The matrix G of the system of equilibrium equations contains the direction cosines of the forces, which are calculated from the coordinates of the nodes:

```
> for i to m do
>      Lxy[1]:=x[N[i][2]]-x[N[i][1]]:
>      Lxy[2]:=y[N[i][2]]-y[N[i][1]]:
> L[i]:=sqrt(Lxy[1]^2+Lxy[2]^2);
> for j to 2 do
>    jj:=2*N[i][2]-2+j:
>    if jj<=m0 then G[jj,i]:=-Lxy[j]/L[i]:fi;
>    jj:=2*N[i][1]-2+j:
>    if jj<=m0 then G[jj,i]:= Lxy[j]/L[i]:fi;
> od;
> od:
```

The Maple system provides a solution in symbolic form by the inverse matrix method. The inverse matrix is found both in the elementary algebra 1 / G. This allows us to obtain a finite expression for the deflection according to the Maxwell-Mora formula

$$\Delta=\sum_{i=1}^{m-3} S_i s_i l_i/(EF),$$

where S_i – the forces in the rods of the farm from the action of distributed load, s_i – the forces from a single vertical load in the middle node of the upper belt, l_i– the length of the rods, E – the modulus of elasticity, F – the cross-sectional area of the rods. The sum does not include three support rods, which are adopted rigid. The successive calculation of trusses with different number of panels reveals first of all that for determinants with a number k multiple of 3 the determinant of the system of equations degenerates. The truss turns out to be kinematically variable. A similar effect of kinematic variability was previously revealed in the spatial truss [4] and flat trusses with a similar lattice [5]. We will calculate for trusses with the number k of panels 1, 2, 4, 5, 7, 8, ... or by a number defined by the formula $k=(6j-(-1)^j-3)/4,\ j=1,2,...$ Thus, for a change in j, the sequence k runs through natural numbers, except for multiples of 3. The general form of formula for the sagging of farms with a different number of panels is distinguished only by the values of the coefficients:

$$EF\Delta_j = P\frac{A_j a^3 + C_j c^3 + H_j h^3}{h^2},$$

where $c=\sqrt{a^2+h^2}$. It is this feature (property of regularity) that allows us to generalize the solution to an arbitrary number of panels. We can add many examples of farms that do not possess this property, and for which general formulas (at least exact ones) can not be inferred or very difficult. Such farms, for example, are trusses with a slope of the upper belt.

The truss studied is regular. By induction we obtain the following expressions for the coefficients

$$A=(4j^3+2(1-(-1)^j)j^2+(24-22(-1)^j)j-17(-1)^j+17)/4,$$
$$B=4(4-3(-1)^j),$$
$$C=((30-24(-1)^j)j+57-57(-1)^j)/4.$$

Thus, by induction with the application in the Maple system, formulas for the deflection of the truss are derived. These solutions can be used to assess the accuracy and reliability of numerical results obtained in specialized packages of structural mechanics [1]. The presence of cases of kinematic degeneracy of the structure under study warns practical engineers and designers about the possibility of similar situations in other systems. For flat trusses, analytical solutions are obtained in [6-13], for spatial ones in [4, 14]. A survey of analytical solutions for flat trusses based on the same approach is contained in [15, 16].

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