УДК: 624.24

The derivation of the formula for arch deflection by the method of double induction in the Maple system

N.A. Belyankin, A. Y. Boyko, M.N. Kirsanov

1)belankin2@gmail.com; Национальный исследовательский университет «МЭИ»

2) trianglukator@mail.ru; Национальный исследовательский университет «МЭИ»

3) c216@ya.ru; Национальный исследовательский университет «МЭИ»

Аннотация

The truss with the cross-shaped lattice of the U-shape has two fixed supports. The analytical dependence of the deflection of the truss is derived from the number of panels in the middle part and the number of panels in the side parts. Operators of the Maple system were used to compile and solve recurrent equations.

Keywords: truss, deflection, Maple, induction.

The method of induction on one parameter using the computer mathematics system Maple [1] is widely used for obtaining analytical dependencies in the problems of the deflection of rod systems. In [2–7] polynomial solutions for the number of panels were obtained for flat trusses, in [8–11] — for flat lattice designs. Spatial trusses from this point of view were studied in [12–14]. In this paper we consider the construction of natural images that includes two natural parameters that regulate its dimensions. Hinged core structure (Fig. 1) is a statically determinate truss. In General, the vertical lateral parts of the m panels and the horizontal middle part — $2n$ panels. The total number of cores in the truss, $N = 8(n + m + 1)$ number of nodes $4(n + m + 1)$. Derive formula for deflection of this structure under the action of a concentrated force P at Midspan.

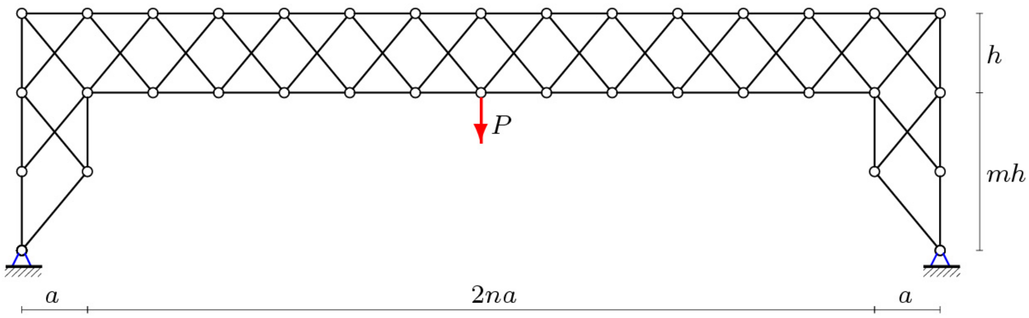

Рис. 1: Truss, $n = 6$, $m = 2$

To determine the stresses in the bars will use the program [1] written in systems of symbol mathematics Maple. The program incorporates a method of cutting of knots. In the original these programs are introduced the coordinates of the nodes and vectors that specify the order of connection of nodes and cores in the same way as in discrete mathematics a graph is defined by edges and vertices. Next, create a matrix ***G*** of the system of equilibrium equations of nodes, consisting of the guides of the cosines of the rods, which in turn are calculated using the coordinates of the nodes. Two fixed supports are modeled by rods whose lengths can be chosen arbitrarily, since the supporting rods are assumed to be rigid. It should be noted that a feature of the proposed design is that the traditional calculation scheme of trusses are not held. The calculation usually begins with the determination of reactions of supports. However, to determine the four reactions of supports of the three equilibrium equations of the whole structure in General is impossible. The method of partitioning in part by analogy with the solution of problems in compound structures, where there is the same problem here also out. The reaction in this case you can define together with the efforts of all the rods from the solution of the global system of equilibrium equations for all nodes. The system of equations ***GS=B*** is solved analytically using the built-in operators of the system Maple.

The method of inverse matrix is used, the more that it is implemented in Maple is very simple. The inverse matrix ***G1*** is calculated by simple division: ***G1:=1/G***. the Vector of the right part of the system of equations associated with external loads. In odd-numbered items are horizontal loads (in this problem they are not), even vertical ***B*[2*(*n+m*)]:=1**. The solution is obtained by multiplying the inverse matrix by the vector of loads **S1:=G1.B.** To determine the deflection using the formula of Maxwell - Mohr $\Delta = P\sum_{i=1}^{N-4} S_i^2 l_i/(EF)$ to calculate the deflection. Here EF is the stiffness of the rods, S_i – stress in the rods from the action of a unit vertical force applied at the middle node, l_i is the length of the rods. Four rigid support rods are not included in the sum. We consider the case of a fixed number of panels in the lateral parts of the truss, $m = 2k = 4$. Consistent calculation of trusses with n=1,2,... 12 showed that the equation for the deflection has the form

$$EF\Delta = P(A_n a^3 + C_n c^3 + H_n h^3)/(2h^2), \tag{1}$$

where $c = \sqrt{h^2 + a^2}$. The method of induction with the involvement of operators **rgf_findrecur** and **rsolve** package genfunc we obtain the total members of the sequence of the coefficients:

$$A_n = [4n^3 - 15n^2 + 27(-1)^n n^2 + 83n - 69(-1)^n n]/6 - 8(-1)^n + 9,$$

$$C_n = 11 - 10(-1)^n + n, \quad H_n = (53 - 43(-1)^n)/2.$$

The most difficult for inductive inference was the coefficient of a^3. The equation for this coefficient has the form

$$A_n = A_{n-1} + 3A_{n-2} - 3A_{n-3} - 3A_{n-4} + 3A_{n-5} + A_{n-6} - A_{n-7}.$$

Figure 2 shows the curves of the dependences for a fixed span of L=2an=100 m. Indicated dimensionless deflection $\Delta' = \Delta EF/(PL)$. Noticeable surges trough and a marked dependence on the height h.

The reaction of supports are also obtained in a symbolic form by induction from the solution of the General system of equilibrium equations for all nodes. Horizontal reaction depends on the parity of n: $X_A = Pa((-1)^n - 1)/(4h)$, the vertical is constant $Y_A = P/2$. In addition, as shown by the bill, these decisions do not depend on the number m of the side panels.

The resulting solution can be generalized to an arbitrary number of panels in the side (supporting) parts of the truss (Fig. 3). As a result of induction on the parameter $m = 2k$, we

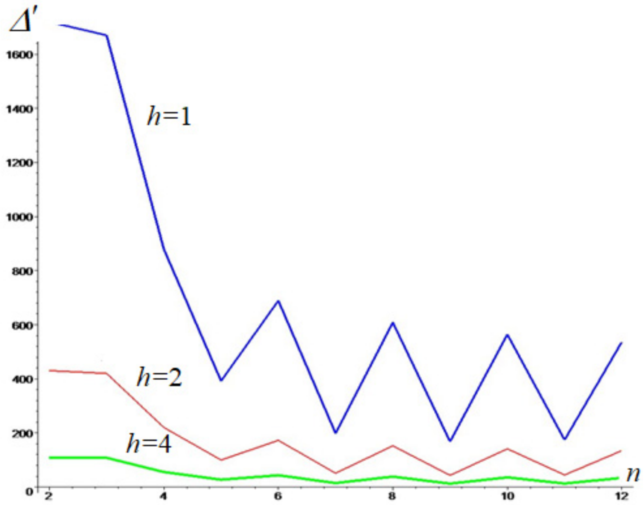

Рис. 2: The deflection- number of panels dependence, $m = 4$

have the following formulas

$$A_n = (4n^3 + [(-1)^n(12k+3) - (12k-9)]n^2 +$$
$$+[24k^2 - 12k + 11 - (24k^2 - 12k - 3)(-1)^n]n \ \ + 12k^2(1 - (-1)^n) + 6)/6,$$
$$H_n = (16k^3 + 14k + 3 - (2k - 3 + 16k^3)(-1)^n)/3,$$
$$C_n = 2k^2 + k + 1 + n - (2k^2 + k)(-1)^n.$$

A review of papers on the topic of analytical calculations in the calculation of flat trusses is contained in [15].

Список литературы

[1] Кирсанов М.Н. Решебник. Теоретическая механика/ М.Н. Кирсанов /Под ред. А.И.Кириллова -М.: Физматлит, – 2008. 382 с.

[2] Астахов С.В. Вывод формулы для прогиба внешне статически неопределимой плоской фермы под действием нагрузки в середине пролета / С.В. Астахов // Строительство и архитектура. –2017. – Т. 5. – № 2. – С. 50–54.

[3] Кирсанов М.Н. Аналитический расчет прогиба распорной фермы с произвольным числом панелей / М.Н. Кирсанов// Механизация строительства. – 2017. – № 3 (873). – С. 26–29.

[4] Кирсанов М.Н. Аналитический расчет прогиба двухпролетной плоской фермы / М.Н. Кирсанов// Механизация строительства. – - 2017. – № 5. – С. 35–38.

[5] Кирсанов М.Н. Аналитический расчет плоской регулярной фермы с растянутыми раскосами / М.Н. Кирсанов// Строительная механика и расчет сооружений. – 2017. – 3(272). – С. 31–35.

[6] Кирсанов М.Н. Статический анализ и монтажная схема плоской фермы/ М.Н. Кирсанов // Вестник государственного университета морского и речного флота им. адмирала С.О. Макарова. – 2016. – № 5 (39). – С. 61–68.

[7] Кирсанов М.Н. Аналитический расчет балочной фермы с решеткой типа "Butterfly"/ М.Н. Кирсанов // Строительная механика и расчет сооружений. – 2016. – № 4. – С. 2–5.

[8] Кирсанов М.Н. Формулы для расчета прогиба и усилий в решетчатой ферме/ М.Н. Кирсанов // Механизация строительства. – 2017. –4(874). – С.20–23.

[9] Кирсанов М.Н., Маслов А.Н. Формулы для расчета прогиба балочной многорешетчатой фермы/ М.Н. Кирсанов // Строительная механика и расчет сооружений. – 2017. – 2(271). – С. 4–10.

[10] Кирсанов М.Н. К выбору решетки балочной фермы / М.Н. Кирсанов // Строительная механика инженерных конструкций и сооружений. – 2017. – N3. – С.23–27.

[11] Кирсанов М.Н. Вывод формулы для прогиба решетчатой фермы, имеющей случаи кинематической изменяемости / М.Н. Кирсанов // Строительная механика и конструкции. – 2017. – № 1 (14). – С. 27–30

[12] Кирсанов М.Н. Аналитическое исследование жесткости пространственной статически определимой фермы / М.Н. Кирсанов // Вестник МГСУ. – 2017. – Т. 12. – Вып. 2 (101). – С. 165–171.

[13] Кирсанов М.Н. Анализ прогиба фермы пространственного покрытия с крестообразной решеткой / М.Н. Кирсанов // Инженерно-строительный журнал. – 2016. – № 4 (64). – С. 52–58.

[14] Кирсанов М.Н. Оценка прогиба и устойчивости пространственной балочной фермы / М.Н. Кирсанов // Строительная механика и расчет сооружений. – 2016. – № 5 (268). – С. 19–22.

[15] Тиньков Д.В. Сравнительный анализ аналитических решений задачи о прогибе ферменных конструкций / М.Н. Кирсанов // Инженерно-строительный журнал. – 2015. – №5(57). – С. 66–73.