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АНАЛИТИЧЕСКИЕ ВЫРАЖЕНИЯ ЧАСТОТ МАЛЫХ КОЛЕБАНИЙ БАЛОЧНОЙ ФЕРМЫ С ПРОИЗВОЛЬНЫМ ЧИСЛОМ ПАНЕЛЕЙ

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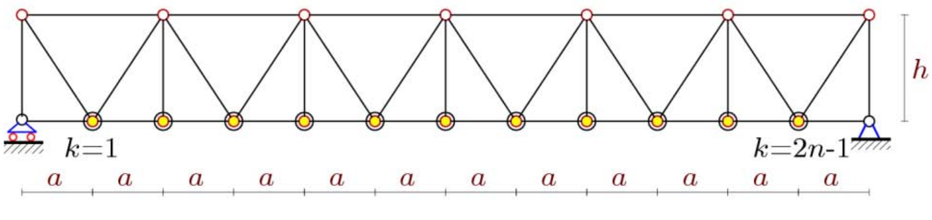
Для вывода аналитической оценки нижней частоты собственных колебаний плоской статически определимой фермы рассмотрена инерционная модель фермы с массами, сосредоточенными в узлах ее нижнего пояса. Смещения грузов предполагаются вертикальными. Прогибы фермы под действием сосредоточенных сил, приложенных к узлам с массами и рассчитанные по формуле Максвелла – Мора, дают значения коэффициентов матрицы податливости фермы. Для оценки по методу Донкерлея требуются только диагональные элементы матрицы. Искомая оценочная формула получена методом индукции расчетом нижней границы первой частоты для отдельных ферм с последовательно увеличивающимся числом панелей. Это дает найти зависимость частоты колебаний фермы не только от ее размеров, но и от числа панелей. Коэффициенты формулы определяются из решения рекуррентных уравнений для элементов последовательностей, полученных из частных решений. В расчетах и анализе использована система компьютерной математики Maple.

Ключевые слова: ферма, первая частота колебаний, оценка Донкерлея, индукция, аналитическое решение,Maple.

The natural oscillation frequencies of the trusses required to assess the dynamics of the structure are calculated, as a rule, numerically [1-6]. In most cases, these solutions are used in optimization problems [7-9]. At the same time, a common and quite acceptable approximation is the truss model with concentrated masses at the nodes [10,11]. However, as practice shows [11-16], in such a formulation it is possible to obtain an analytical solution. The most demanded in practical calculations is the first, the lowest frequency of oscillations. The Donkerley method [17] makes it possible to derive a fairly simple formula for the lower bound of this value.

Consider a truss with a triangular lattice consisting of n identical panels (Fig. 1). Each panel includes two bars of lower chord of length a, two multidirectional struts of length $c=\sqrt{a^2+h^2}$ and struts of height h. In the truss the $m=3n+5$ bars, including three bars corresponding to the supports.



Rice. 1. Trussat n=6. In the nodes of the lower chord are the masses m_k, $k=1,...,2n-1$

The mass of the truss is evenly distributed over the nodes of the lower chord of the truss: $m_k = m,\ k = 1,...,2n-1$. The estimation formula of Dunkerley gives a lower bound of the first natural frequency $\tilde{\omega}$ of the mass in the node

$$\tilde{\omega}^2 = 1 / \sum_{k=1}^{2n-1} 1/\omega_k^2 \ . \qquad (1)$$

Here is a notation ω_k for the oscillation frequency of a single mass m_k, at the node k + 1 of the lower chord, in the absence of all other masses. Numbering of knots is conducted from the left support. The differential equation of mass $m_k = m$ oscillations of the second order has the form

$$m\ddot{y}_k + d_{k,n} y_k = 0,$$

where y_k — vertical mass displacement, $\ddot{y}_k$ — acceleration, $d_{k,n}$ — stiffness coefficient of mass with number k, n— number of panels in the truss. It follows that the frequency of oscillations of the load $\omega_k = \sqrt{d_{k,n}/m}$. The stiffness coefficient can be calculated through the flexibility coefficient by the Maxwell-Mohr formula:

$$\delta_{k,n} = 1/d_{k,n} = \sum_{i=1}^{m-3} \frac{S_{i,k}^2 l_i}{EF_i} .$$

Here it is indicated $S_{i,k}$ — the forces in the bar numbered $i = 1,...,m-3$ from the action of a single vertical force applied to the node in which the load is located with the number k , l_i — the length of the bar. It is accepted that the stiffness of all bars is the same: $EF_i = EF$. Three rigid support bars are not included.

The forces in the bars of the truss are determined by the program [11] in symbolic form. In the program by analogy with [11-17] coordinates of nodes and the scheme of connection of bars in nodes are entered. The origin is located in the left support:

$$x_i = a(i-1),\ \ y_i = 0,\ i = 1,...,2n+1,$$
$$x_{i+2n+1} = 2a(i-1),\ \ y_{i+2n+1} = h,\ \ i = 1,...,n+1.$$

The lattice of the truss and its chord is formed by vectors $\bar{N}_i,\ i = 1,...,m$ containing the numbers of the ends of the bars. The upper and lower chords are defined as follows:

$$\bar{N}_i = [i, i+1],\ \ i = 1,...,2n,$$
$$\bar{N}_{i+2n} = [i+2n+1, i+2n+2],\ i = 1,...,n.$$

The grid (struts and braces) correspond to the vectors

$$\bar{N}_{i+3n+1} = [2i-1, i+2n+1],\ i = 1,...,n+1,$$
$$\bar{N}_{i+4n+1} = [2i, i+2n+1],\ \bar{N}_{i+5n+1} = [2i, i+2n+2],\ i = 1,...,n.$$

The system of equilibrium equations of nodes in projections on the axis is entered into the program written in matrix form. The matrix of the system consists of the guiding cosines of the forces. Odd rows of the matrix consist of projections of unit vectors of forces on the horizontal axis, even — on the vertical. Depending on the considered node with mass, to which a single force is applied, the distribution of forces on the bars is markedly different depending on the parity of the load number (Fig. 2,3). Compressionbars are highlighted in black, tension – in red. The thickness of the lines is proportional to the force.

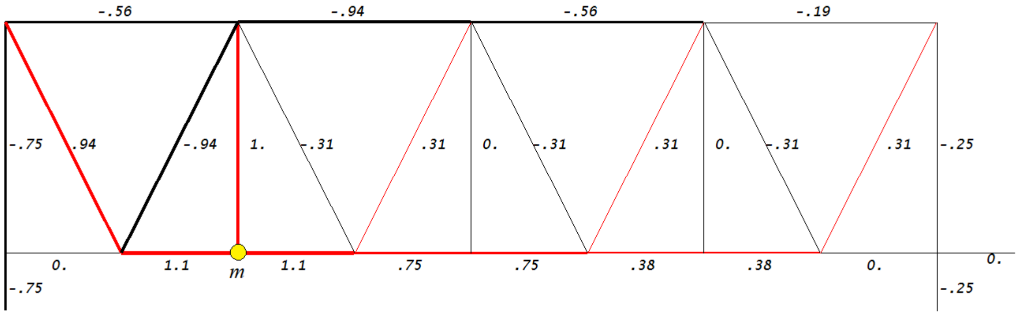


Fig. 2. The distribution of forces in the bars in fractions of 1 when considering the mass with an even number n=4, k=2, a=3m, h=4m

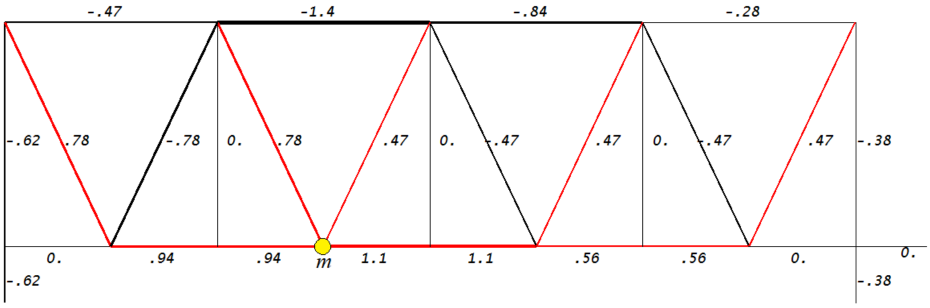


Fig.3. The distribution of stresses in the bars when considering odd weight
n=4, k=3, a=3m, h=4 m

Solving individual problems for trusses with a consistently increasing number of panels n=1, 2, 3,..., gives an expression of the flexibility coefficient from the action of a unit force in a node with mass k of the form

$$\delta_{k,n}=\frac{C_{1,k,n}a^3+C_{2,k,n}c^3+C_{3,k,n}h^3}{n^2h^2EF},$$

To obtain the dependence of the coefficients in this expression on the number of panels n and the number of nodes with masses k, the induction method is used. First, with a fixed number of nodes with masses coefficients are obtained for a different number of panels:

$$\delta_{1,1}=\frac{a^3+c^3+h^3}{2h^2EF},\ \delta_{1,2}=\frac{14a^3+6c^3+5h^3}{8h^2EF},\ \delta_{1,3}=\frac{55a^3+15c^3+13h^3}{18h^2EF},$$
$$\delta_{1,4}=\frac{140a^3+28c^3+25h^3}{32h^2EF},\ldots$$

For the coefficient $C_{1,1,n}$ using the operator **rgf_findrecur** of the Maple system we obtain a recurrent equation for the common term

$$C_{1,1,n}=4C_{1,1,n-1}-6C_{1,1,n-2}+4C_{1,1,n-3}-C_{1,1,n-4}.$$

The solution of this equation can be obtained using the **rsolve** operator:

$$C_{1,1,n}=4n^3/3-n^2+n/6.$$

Similarly, for k=1, we find the coefficients

$$C_{2,1,n} = n^2 - n/2,$$

$$C_{3,1,n} = n^2 - n + 1/2.$$

When k=2 we have

$$C_{2,1,n} = 16n^3/3 - 10n^2 + 14n/3,$$

$$C_{2,1,n} = 2n^2 - 2n,$$

$$C_{3,1,n} = 2n^2 - 2n + 2.$$

To obtain a generalization of the number of masses$k$ will be required to carry out the calculations for k=1,2,...8. Generalizing these solutions using the same operators **rgf_findrecur**and **rsolve**, we obtain the final formula for the coefficients

$$C_{2,k,n} = nk(2k^2 - 4nk - 1)(k - 2n)/6,$$
$$C_{2,k,n} = kn(2n - k)/2,$$
$$C_{3,k,n} = (((-1)^k + 3)n^2 - 2nk + k^2)/2.$$

Thus, the dependence of the flexibilitycoefficient on the number of panels and number of nodes with mass is obtained. For even n = 2j, the expression has the form

$$\delta_{k,n} = (z_1 a^3 + z_2 c^3 + z_3 h^3)/(6n^2 h^2 EF),$$

where

$$z_1 = 2jk(4j - k)(1 + 8jk - 2k^2),$$
$$z_2 = 6jk(4j - k),$$
$$z_3 = 3(k^2 - 4jk + 4j^2(-1)^k + 12j^2).$$

Summing, according to $\omega_k^2 = 1/(m\delta_{k,n})$ (1), we obtain the desired lower estimate of the first frequency of natural oscillations.

$$\tilde{\omega}_n = 6h\sqrt{5jEF/(m(\ 2j(512j^4 + 80j^2 - 7)a^3 + 30(16j^2 - 1)jc^3 + 15(1 - 24j + 56j^2)h^3\ ))}\ .$$

A similar but more cumbersome expression holds for odd n's.

Graphs of the dependence of the oscillation frequency on the span length show that at a fixed span length, an increase in the number of panels leads to a decrease in frequency (Fig. 4, 5).

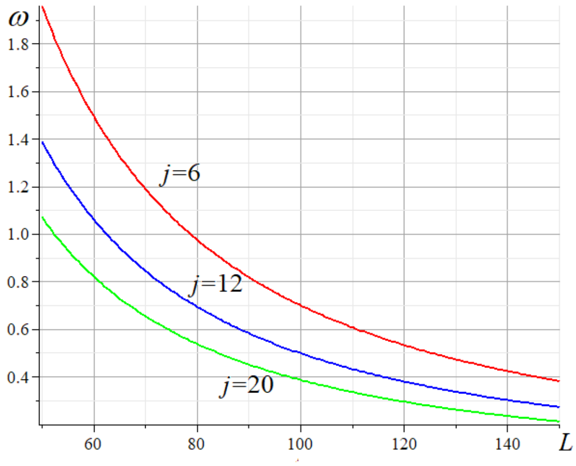


Fig. 4. Oscillation frequency depending on span length and number of panels at h=4m, $a = L/n = L/(2j),\ EF = 2\cdot 10^4 kN,\ m = 150\,kg$

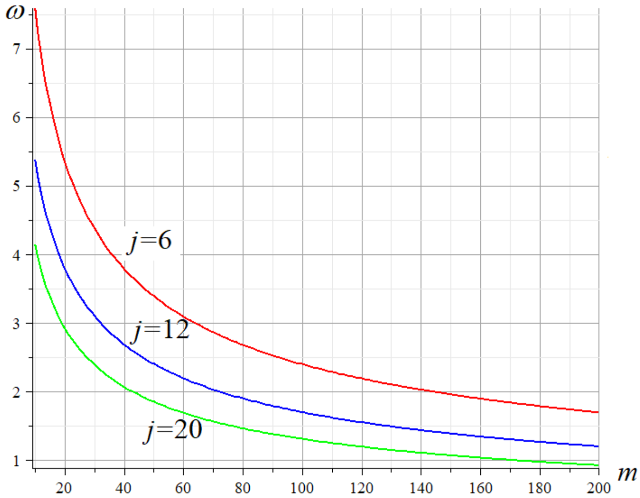


Fig. 5. Oscillation frequency depending on span length and number of panels at *h*=4m,
$a = L/n = L/(2j),\ L = 50m,\ EF = 2\cdot 10^4 kN$

In conclusion, it is stated that in comparison with solutions with one natural parameter specifying the order of the regular structure, to which we apply the induction method when deriving the general solution [18], in problems of vibration of a system with a discretely distributed mass (here - at the nodes of the lower chord) at least two natural parameters arise - the number of panels and the node number with mass. This greatly complicates the task. For example, if 10 separate solutions are required to obtain a sequence of numbers long enough to reveal its common term, then in a two-parameter problem this number increases to about 100. It should be borne in mind that symbolic transformations in computer mathematics systems require an order of magnitude more time than numerical transformations. Therefore, it is not always possible to construct an analytical dependence of dynamic characteristics on the order of a regular system.

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ANALYTICAL EXPRESSIONS OF FREQUENCIES OF SMALL OSCILLATIONS OF A BEAM TRUSS WITH AN ARBITRARY NUMBER OF PANELS

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To derive an analytical estimate of the lower eigenfrequency of a plane statically determinate truss, an inertial model of a truss with masses concentrated in the nodes of its lower chordis considered. Displacements of the nodes with masses are assumed to be vertical. The deflections of the truss under the action of concentrated forces applied to the nodes with masses and calculated by the Maxwell-Mohr formula give the values of the coefficients of the truss flexibility matrix. For the evaluation according to the method of Dunkerley only requires the diagonal elements of the matrix. The required estimate formula is obtained by induction calculation of the lower bound of the first frequency for individual trusses with a consistently increasing number of panels. This makes it possible to find the dependence of the frequency of oscillations of the truss not only on its size, but also on the number of panels. The coefficients of the formula are determined from the solution of recurrent equations for elements of sequences obtained from partial solutions. Maple computer mathematics system is used in calculations and analysis.

Keywords: truss, the first frequency of oscillation, assessment of Dunkerley, induction, analytical solution, Maple