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ANALYTICAL DEPENDENCE OF THE LOWER BOUND OF THE NATURAL OSCILLATION FREQUENCIES OF THE MANIPULATOR TRUSS FROM THE NUMBER OF PANELS

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An analytical estimation of the fundamental frequency of oscillations of a planar statically determinate model of a manipulator with an arbitrary number of panels in the console part and rack is derived by the method of induction using two parameters with the involvement of operators of the Maple computer mathematics system. The mass is allocated to the nodes of the lower belt of the console. To output the formula, the operators of the specialized LinearAlgebra and genfunc packages are used. The coefficients in the desired formula are found as solutions of recurrent equations composed according to a series of solutions for trusses with a consistently increasing number of panels. Compared to similar solutions for planar regular trusses that use Dunkerley estimation, the solution is distinguished by its high accuracy. Numerical analysis of the truss oscillation spectrum, taking into account many degrees of freedom, shows that the error of the obtained analytical estimate does not exceed 2% on average .

Key words: truss, console, method of Dunkerley, the oscillations of lower frequency, induction, Maple.

Introduction

The calculation of manipulators designed for high-speed work involves the analysis of natural oscillation [1]frequencies. One of the most significant dynamic characteristics of a structure is its first (lowest) natural frequency. If the structure contains a large number of elements (rods), the complete calculation of a system with many degrees of freedom (based on the number of nodes endowed with masses) is a complex numerical problem. The development of modern systems of symbolic mathematics makes it possible to use analytical methods along with numerical methods. The value of such solutions is determined by both accuracy and versatility. An analytical solution in the form of a formula obtained for a single structure with parametrically specified dimensions and loads has significantly less value than a solution that takes into account the number of elements. For regular structures with periodically repeated elements, it is possible to take into account their number in the final calculation formula using the induction method [1]. In this paper, we consider a planar, statically determinate model of the truss of a cantilever manipulator with a mass distributed over the nodes of the lower belt of the console. The solution is based on the method used earlier in the problems of analyzing the static deflection of trusses [1]. The frequencies of beam truss oscillation were studied analytically in [2–10].

Solution

The truss post (Fig. 1) with two supports has m cross-shaped panels in height. The console contains n panels with a triangular grid. In total, there are $\eta = 4(m+n)+6$ rods in the truss, counting three rods that model movable and fixed supports. Ignoring the horizontal displacements, we consider only the vertical oscillations of the loads in the nodes of the structure. The number of degrees of freedom of such a model is equal to the number of loads $N = n$. The truss diagram has a regular type with two parameters n and m. The stress state of the truss rods can be calculated analytically using the program in the Maple system [1, 4–7] . The program includes a method for cutting out nodes.

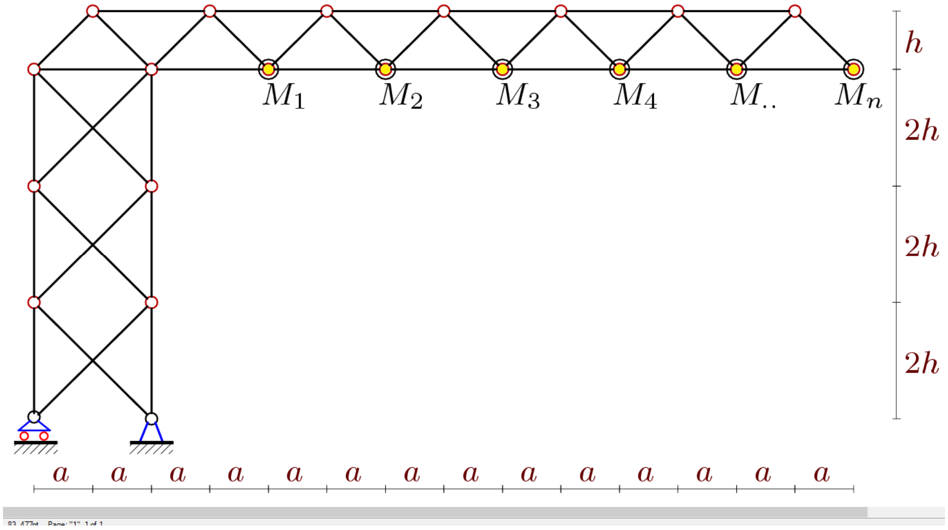

Fig. 1. The scheme of the manipulator, m=2, n=5

The equations of mass oscillations have the form:

$$\mathbf{J}_N \ddot{\mathbf{Y}} + \mathbf{D}_N \mathbf{Y} = 0, \tag{1}$$

where $\mathbf{D}_N$ is the stiffness matrix, $Y = [y_1, y_2, ... y_N]^T$ is the vector of vertical displacements of loads. If all the masses are equal $M_1 = M_2 = ... = M_n = M$, then $\mathbf{J}_N = M\,\mathbf{I}_N$ – is the diagonal inertia matrix, $\mathbf{I}_N$ – the unit matrix, $\ddot{\mathbf{Y}}$ – the acceleration vector of nodes with masses. The inverse of the stiffness matrix $\mathbf{D}_N$ is the matrix $\mathbf{B}_N$, whose elements are calculated using the Mohr's integral:

$$b_{i,j} = \sum_{k=1}^{\eta-3} S_k^{(i)} S_k^{(j)} l_k / (EF). \tag{2}$$

Here, $S_k^{(i)}$ is the force in the rod k from the action of a single vertical force at the node i, l_k is the length of the rod k, E is the elastic modulus of the rod material, F is the cross-sectional area of the rods. The stiffness of the rods is assumed to be the same. The three support rods are not deformed. The forces in these rods are not included in the sum (2).

The approximate solution according to the Dunkerley method [8] for the lower estimate of the first oscillation frequency ω_D is expressed in terms of the oscillation frequencies of individual loads in the marked nodes:

$$\omega_D^{-2} = \sum_{k=1}^{N} \omega_k^{-2}, \tag{3}$$

where ω_k is the partial oscillation frequency of the mass M located in the console node on the lower belt. In the case of oscillations of one mass, equation (1) has the form: $M\ddot{y}_k + d_k y_k = 0$, where d_k is the coefficient of rigidity, y_k is the displacement of the mass, and $\ddot{y}_k$ is the acceleration. Hence, the oscillation frequency of a single load (partial frequency) has the form: $\omega_k = \sqrt{d_k / M}$. The stiffness coefficient is calculated using the Mohr integral: $\delta_k = 1/d_k = \sum_{j=1}^{\eta-3} \left(\tilde{S}_j^{(k)}\right)^2 l_j / (EF)$. Here it is indicated: $\tilde{S}_j^{(k)}$ – the forces in the rod with the number j from the action of a single vertical force applied to the node where the mass with the number k is located . From (3) follows:

$$\omega_D^{-2} = M\sum_{k=1}^{N}\delta_k = M\Delta_n. \quad (4)$$

General view of the solution for the Δ_n coefficient at $m = 3$:

$$\Delta_n = (C_{1,n}a^3 + C_{2,n}c^3 + C_{3,n}h^3)/(h^2 EF) \quad (5)$$

Solving the problem sequentially for $n = 1, 2, 3, ...$, we get:

$$\Delta_1 = 2(15h^3 + 6a^3 + 2c^3)/(h^2 EF), \qquad \Delta_2 = 4(27h^3 + 20a^3 + 4c^3)/(h^2 EF),$$

$$\Delta_3 = 2\left(129h^3 + 140a^3 + 20c^3\right)/\left(h^2 EF\right),\ \Delta_4 = 8\left(63h^3 + 90a^3 + 10c^3\right)/\left(h^2 EF\right),$$

$$\Delta_5 = 10\left(87h^3 + 90a^3 + 14c^3\right)/\left(h^2 EF\right),\ \ \Delta_6 = 4\left(345h^3 + 728a^3 + 56c^3\right)/\left(h^2 EF\right),...$$

Using the **rgf_findrecur** operator from the special **genfunc** package of the Maple system, you can get recurrent equations for sequence elements. For the coefficient C_1, we have a fifth-order linear equation: $C_{1,n} = 5C_{1,n-1} - 10C_{1,n-2} + 10C_{1,n-3} - 5C_{1,n-4} + C_{1,n-5}$.

The **rsolve** operator gives a solution to this equation:

$$C_{1,n} = (4n^4 + 14n^3 + 14n^2 + 4n)/3\,. \quad (6)$$

Other coefficients are found in the same way:

$$C_{2,n} = 4n^3 + 12n^2 + 14n,\ C_{3,n} = (2n^3 + 4n^2 + 4n)/3. \quad (7)$$

To generalize the solution to an arbitrary number, you need to repeat the entire solution sequentially for different $m = 1, 2, ..$ Calculations show that only the coefficient C_2 depends on the number of panels m vertically, and then linearly. Thus, we have the values of the coefficients in the general case

$$C_1 = 2n(2n+1)(n+2)(n+1)/3,\ C_2 = 2m(n+1)(2n^2+4n+3)/3,\ \ C_3 = 2n(n+1)(n+2)/3.$$

Hence, taking into account (4) and (5), we obtain the final formula for the lower bound of the first natural frequency of oscillation of the truss:

$$\omega_D = h\sqrt{\frac{3EF}{2M(n+1)(n(n+2)((2n+1)a^3+c^3)+m(2n^2+4n+3)h^3)}}\ . \quad (8)$$

Analysis of the obtained results

The accuracy of analytical solutions (4) factors (6, 7) can be estimated from a comparison with the solution of the problem of oscillation of a system with many degrees of freedom, N, obtained numerically. To find the eigenvalues of a matrix $\mathbf{B}_N$, we use the **Eigenvalues** operator from the LinearAlgebra package of the Maple system. The graph (Fig. 2) shows the curves of the dependence of the first frequency ω_{num} obtained numerically and ω_D according to the formula (4). The curves are almost identical. Accepted: $EF = 1000H$, M = 100kg, a = 3m, h = 4m. Accuracy (relative error) $\varepsilon = \left(\omega_{num} - \omega_D\right)/\omega_{num}$ depends on the number of panels (fig. 3).

The resulting formula can be used to estimate the frequency of oscillation of the truss with a very large number of rods. As you know, the accuracy of the numerical calculation decreases with an increase in the number of structural elements, while the analytical solution has an almost constant and high accuracy. This can be seen from the curve in Figure 3, which goes to the horizontal asymptote.

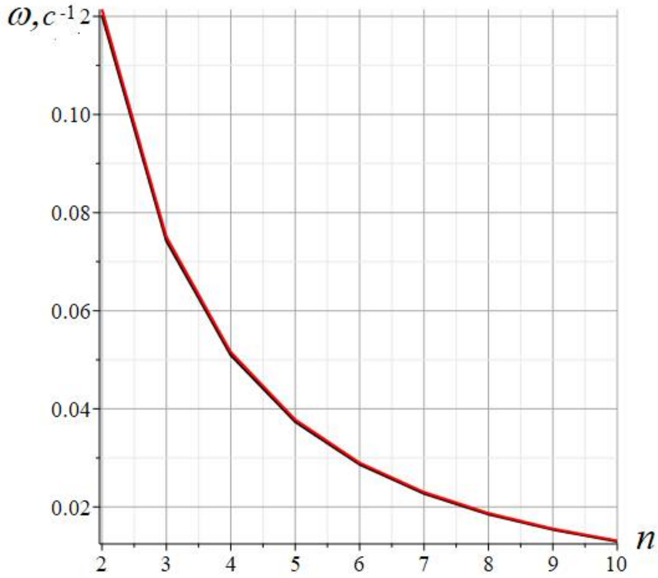

Fig. 2. Frequency dependence on the number of panels

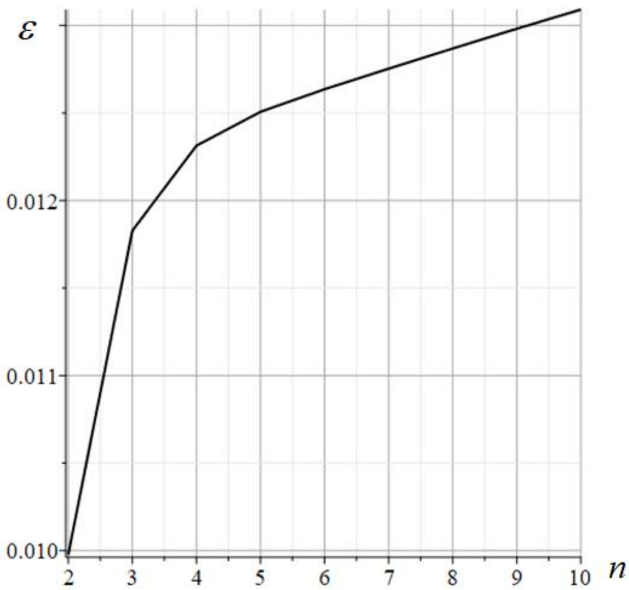

Fig. 3. The error of Dunkerley's estimation depending on number of panels

The error of the solution varies depending on the number of panels from 1%, for $n = 2$, to 1.3% for a large number of panels. For comparison, we note that in [11] in the problem of vibrations of the nodes of a beam truss with a triangular lattice and with two panels, the solution according to the Dunkerley method gives an error of 29%.

The oscillation frequency depends non-linearly on the height of the panel h (Fig. 4). The graphs are constructed according to the analytical solution (8) at $m = 7$ and the same values of mass and stiffness as the previous graphs. As the number of n panels in the console increases, the maximum frequency value shifts to the right on the graph.

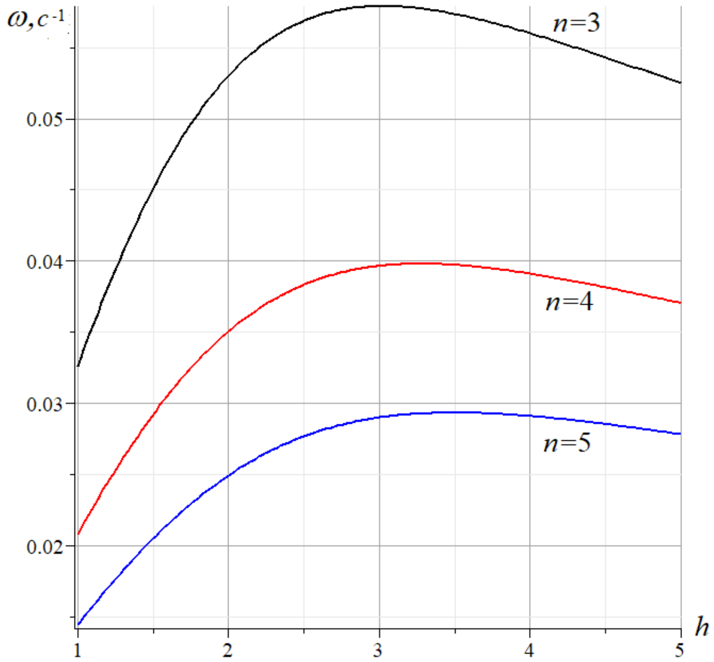

Fig. 4. Dependence of the oscillation frequency on the height of the truss

Another method for obtaining an analytical estimate of the lowest frequency is based on the Rayleigh method [11]. This method gives even greater accuracy and estimates the frequency from above, but the resulting formula is too cumbersome.

Conclusion

The method of estimating the first frequency by Dunkerley is known, but it is rarely used in practice, since its accuracy is low and in the case of complex systems with many degrees of freedom it involves numerical counting. The development of computer mathematics systems and the method of induction used in the derivation of analytical dependencies for regular systems allowed us to obtain a number of analytical solutions, in particular for the problem of the truss oscillation. The proposed solution turned out to have high accuracy and a very compact shape. The authors still do not know what caused such a high accuracy — whether the choice of the design model, the neglect of horizontal oscillations, or something else. In any case, the formula is quite convenient for use in practice, and the proposed algorithm can be used in solving other similar problems for statically determinate systems.

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АНАЛИТИЧЕСКАЯ ЗАВИСИМОСТЬ НИЖНЕЙ ГРАНИЦЫ СОБСТВЕННОЙ ЧАСТОТЫ КОЛЕБАНИЙ ФЕРМЫ МАНИПУЛЯТОРА ОТ ЧИСЛА ПАНЕЛЕЙ

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Методом индукции по двум параметрам с привлечением операторов системы компьютерной математики Maple выведена аналитическая оценка основной частоты колебаний плоской статически определимой модели манипулятора с произвольным числом панелей в консоли и стойке. Массой наделены узлы нижнего пояса консоли. Для вывода формулы используются операторы специализированных пакетов LinearAlgebra и genfunc. Коэффициенты в искомой формуле находятся как решения рекуррентных уравнений, составленных по серии решений для ферм с последовательно увеличивающимся числом панелей. По сравнению с аналогичными задачами для плоских регулярных ферм, где используется оценка по Донкерлею, полученное решение отличается высокой точностью. Численный анализ спектра колебаний фермы с учетом многих степеней свободы показывает, что погрешность полученной аналитической оценки в среднем не превышает 2% .

Ключевые слова: ферма, консоль, метод Донкерлея, колебания, нижняя частота, индукция, Maple