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Упругие характеристики стохастически неоднородной среды, моделируемой стержневой сеткой

Kirsanov M.N., Kollerova S.E.
Elastic characteristics of a stochastically inhomogeneous medium modeled by a rod grid

Упругая среда изучается по деформациям плоского прямоугольного сеточного образца, составленного из шарнирно соединенных стержней так, что они образуют статически определимую конструкцию. Деформации определяются по формуле Максвелла-Мора. Находятся статистические характеристики модуля упругости среды и коэффициента Пуассона
Ключевые слова: *упругость, неоднородность, стохастическая среда, стержни*

Elastic medium is studied on the deformation of flat rectangular grid pattern, composed of hinged rods so that they form a statically determinate design. Deformation determined by the formula of Maxwell-Mohr. The statistical characteristics of the modulus of elasticity of the medium and of the Poisson's ratio are obtained
Key words: *elasticity, heterogeneity, stochastic environment, rods*

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Stochastic properties of inhomogeneous media have always been under close attention of researchers [1-3]. In [4] considered a stochastic algorithm that uses basic concepts of the theory of elasticity. It is assumed that a physical quantity cannot be defined exactly, and their values are found with some probability. The physical parameters are treated as random variables having appropriate characteristics. In [5] studied the stability of stochastically inhomogeneous, compressible elastic medium in three-dimensional formulation. The external load is deterministic subcritical deformation finite. In problems of dynamics of deformed systems with random properties [6] used Monte-Carlo.

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Consider an elastic model of a stochastically inhomogeneous medium in the form of a rod design with a random arrangement of grid nodes (Fig. 1). To determine the integral characteristics of the environment module of elasticity and Poisson's ratio define the deformation of the sample under the action of uniformly distributed over the nodes of the upper boundary of the load. We have the equation of Maxwell – Mohr used in structural mechanics [7-9] to calculate the displacements in elastic structures. The rods of the model working only in tension and compression, so the formula includes only the longitudinal stress: $\Delta = \sum_{i=1}^{N_s - N_o} S_i s_i l_i / (EF)$. Used notations: S_i –the forces in the rods from the action of external loads, s_i – the forces in the rods from the action of a single force applied to the control point to measure the deflection, l_i – length of rods, N_s – number of rods together with the supporting rods. A rigid support rod in the sum are not included $N_o = 2n$. In the case when the model contains n cells horizontally and m vertically, the number of cores $N_s = 2(mn - m + n)$ and number of nodes (joints) $2mn - m + 3n$. The ratio of the number of joints and number of rods 1:2 meets the necessary condition for static definability of the structure. The calculation of forces will produce using cut nodes in the program [10] written in the Maple language. We give the fragment program, which are specified by the coordinates of the nodes:

```
> for j to m do
> kfx:=2e11*n:# the amplitude of random fluctuations on the axis X
> kfy:=1e11*n: k:=(j-1)*(2*n-1):
> for i to n do
dx:=rand()/kfx: x[i+k]:=a*i-a+dx:
dy:=rand()/kfy: y[i+k]:=(2*j-1)*h-h+dy: od:
> for i to n-1 do
dx:=rand()/kfx: x[i+n+k]:=a*i-a+b+dx:
dy:=rand()/kfy: y[i+n+k]:=(2*j-1)*h+dy: od:
> od:
> k:=m*(2*n-1):
> for i to n do x[i+k]:=a*i-a: y[i+k]:=2*m*h: od:
```

Here **rand()** random 12-digit natural number. The parameters *a*, *h* and *b* determine the size of the grid. In this calculation it is accepted **a:=40/n, h:=20/m, b:=20/n**. The structure of connections of members and nodes of the set of special vectors contain the numbers of the member ends. These vectors in this case are specified in cycles of the height and width of the specimen and have a look

```
> for j to m do
> z:=4*(n-1):p:=2*n-1:
> for i to n-1 do
> N[i+z*(j-1)]:= [i+p*(j-1),i+n+p*(j-1)];
> N[i+n-1+z*(j-1)]:=[i+1+p*(j-1),i+n+p*(j-1)];
> N[i+2*(n-1)+z*(j-1)]:=[i+2*n-1+p*(j-1),i+n+p*(j-1)];
> N[i+3*(n-1)+z*(j-1)]:=[i+2*n+p*(j-1),i+n+p*(j-1)];
> od:od:
```

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When calculating the horizontal deformation using Maxwell – Mohr defines the displacement of the nodes in the middle part of the sample left and right (Fig. 1). For this purpose, the vector of the right part of the system of equilibrium equations of nodes is specified for multiple control points. For points on the right side of the sample are **BRight[2*nRight-1]:=1**, where **nRight:=(m1+1)*n+(n-1)*(m1)** – the number of the corresponding grid points, **m1=2m**. Left points have **BLeft[2*nLeft-1]:=1, nLeft:=nRight-n+1**. Horizontal forces are recorded in the odd lines, the vertical is even. Experience showed that the measurement of elastic properties only at two points does not give sustainable results. In some cases, due to local deflections Poisson's ratio beyond acceptable values [0, 0,5]. Therefore, in the calculation introduced some more control points and corresponding vectors of the right parts. The computational speed with almost no changes, as when solving a system of linear equations of the selected method inverse of a matrix, for which the main time spent for matrix inversion. In the system Maple it looks amazing just: **G1:=1/G**, **SL:=G1.BLeft**. Deformations are calculated by formulas

$$\varepsilon_x = \Delta_x / (na), \varepsilon_y = \Delta_y / (mh).$$

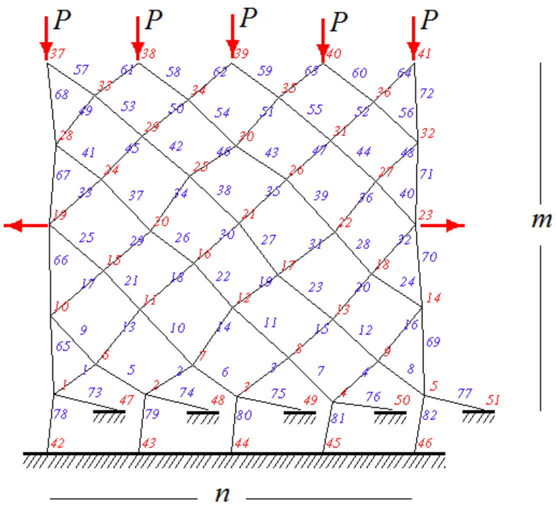

Fig. 1. Core mesh

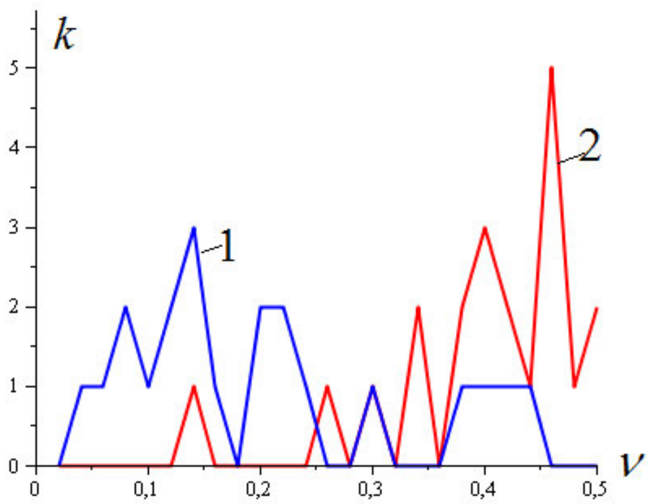

Fig. 2. The distribution of Poisson's ratio

Calculation (test 21) produced when *an*=40m, *hm*=20 m, *bn*=10 m. The curves marked as: 1 – *n*=41, *m*=40 (blue), and 2 – *n* =21, *m*=20. The mathematical expectation of the Poisson's ratio (fig. 2) $\nu_1 = \varepsilon_x / \varepsilon_y = 0{,}19$, $\nu_2 = 0{,}39$ with the variance $\sigma_1 = 0.120$, $\sigma_2 = 0.086$. To calculate the modulus of elasticity of the medium will ask the parameters of the rods of the grid: the cross-sectional area $F = c^2$, $c = 0{,}01м$, modulus of elasticity of the material of rods: $E = 2\cdot10^5$MPa (steel). When *n* =21, *m*=20 obtained the average value of the modulus of elasticity $E_0 =$ 684,7 MPa , with *n* =31, *m*=30 will receive $E_0 = 425{,}9$ MPa . As expected, the mesh material forming the test piece, is much less rigid than the material of the rods of the grid.

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