

Genetic Algorithm for Optimization of Heating Networks

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Abstract – An algorithm for redistribution of flows in an oriented network is proposed, the maximum flow of which is sought by the Push-Relabel method in the Maple system of computer mathematics. At each stage of optimization, the most effective options are identified, the combination of which gives the next generation of genes. It is shown that highly ramified networks are most sensitive to the distribution of the capacity of individual branches. For example, the maximum flows in networks with two and three parallel branches of the same bandwidth are compared. It is shown which parameters of the algorithm have the greatest influence on the result.

Keywords— heat networks, flows, graph theory, Maple, genetic algorithm.

I. INTRODUCTION

Optimization of the throughput capacity of heating networks is of great importance for their efficient and economical operation. The greatest effect of such optimization is achieved when the total network capacity is limited. In the present study, an algorithm for network optimization is proposed based on the Ford-Fulkerson algorithm for determining the throughput of the entire network and a genetic algorithm for distributing the throughput of its branches. The algorithm is implemented in the Maple [1] computer mathematics system. Similar algorithms with different application aspects have been proposed earlier.

Article [2] implements two approaches to the genetic algorithm, distinguishing genetic crossover operator: segmented genetic algorithm (SGA) and an equally efficient Basic Genetic Algorithm (BGA). The authors of the article [3] attempt to optimize the multilayer neural network of the perceptron using several optimization methods to predict the heating and cooling of energy-efficient buildings. Article [4] describes a method combining a lumped-capacitance thermal network model that is effective for accelerating thermal design optimization of a transient heating circuit board layout and Bayesian optimization. For a limited set of buildings, Chi A., Xu Y. [5] proposed a digital gene map characterized by lines of binary code. Based on the digital gene map, building elements were parameterized to create dynamic variables to facilitate a multipurpose genetic algorithm. With the help of a multipurpose genetic algorithm and a special data statistics tool, a "Pareto front" solution was obtained in the work to optimize the decision-making process when designing a series of buildings. An artificial neural network combined with a genetic algorithm is used to optimize the engine thermal strategy [6].

II. METHOD

The input of the algorithm is a directed graph of the network with given bandwidths of the arcs. These data are used to determine the integrated bandwidth of the network.

A significant simplification of the solution was achieved due to the presence of a special MaxFlow operator from the GraphTheory package of the Maple system. The operator uses the Push-Relabel (Push-PreFlow) algorithm developed by Goldberg [7-9]. This is an algorithm similar to the well-known Ford - Fulkerson algorithm [10]. The redistribution of the capacities of the edges (we will conditionally call them diameters) leads to a change in the total flow in the network. Redistribution of diameters in the algorithm is performed by decreasing the diameter of one of the arcs and increasing the diameter of the other arc by the same amount. The arc numbers are chosen randomly. The efficiency of the algorithm significantly depends on the quality of the random number generator. It uses the *rand* () generator of the Maple system. The numbers of arcs with altered diameters form the chromosome of the next generation of genetic evolution. The competition of chromosomes for selection in the creation of the next generation is done simply by choosing the options with the highest flux. At the initial stage, chromosomes $\bar{Z}_k^{(j)} = \{d_{1,k}^{(j)}, \dots, d_{n,k}^{(j)}\}$ are formed, where

$k = 1, 2, \dots, m$, m is the number of chromosomes in a generation (j), n is the number of pairs of changed arcs (chromosome length). The number of chromosomes and their length are related to the size of the network. The values of the genes in the first population are selected using a random number generator. Let be the number of the best chromosome in the population. The first genes in the first chromosome of the next (new) population have the following genes: $\bar{Z}_k^{(j)} = \{d_{1,k}^{(j)}, \dots, d_{n,k}^{(j)}\}$, $k = 1, 2, \dots, m$,

the last genes: $d_{i+n/2,1}^{(j+1)} = d_{i+n/2,\beta}^{(j)}$, where β is the number of the second in quality chromosome of the first generation. The second chromosome is formed according to the same principle, but already according to the data of the chromosomes and the third quality chromosome from the previous generation. The last two chromosomes of the next generation are obtained randomly and give the effect of mutation, which is necessary in such algorithms to eliminate the looping effect after enumerating a finite number of combinations.

III. EXAMPLE

Consider a weighted network of order $N = 18$ and size $M = 32$ (Fig. 1). The directed weighted graph is specified by the operator $G := \text{Digraph}(A, \text{weighted})$. Here is a fragment of a program that implements a genetic algorithm in the language of symbolic mathematics Maple. The problem has a parameter N that sets the order of the graph. Let's choose a network with two parallel branches 2-9 and 10-17 and arcs connecting these branches. It is convenient to set the weights of arcs in cycles. The weight matrix A_0 of arcs (nominal diameters) is taken in the form

```

k:=9: N:=2*k:
for i to k-1 do
  A0[i,i+1]:=2:
od:
for i to k-1 do
  A0[i+k,i+k+1]:=2+i:
od:
for i to k-2 do
  A0[i+k,i+2]:=3:
  A0[i+1,i+k+1]:=1+i:
od:
A0[1,k+1]:=4+i: A0[k,N]:=21-i:

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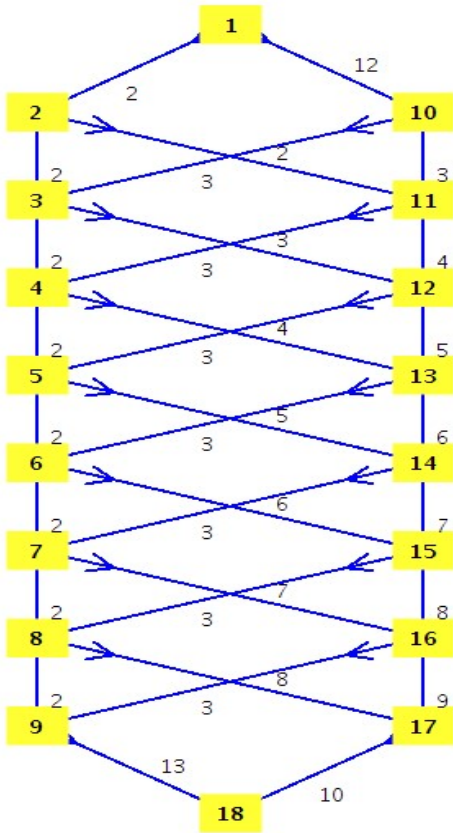


Fig. 1. Planar heat network 1, $N=18$, $M=32$

The weight matrix is asymmetric, since the graph is oriented. Whether the constructed graph is a network is determined by the operator $IsNetwork(G)$. The same operator calculates the numbers of the drain and source vertices in the network. In this case, for any N , the source of the network is vertex 1, and the sink is vertex N . The algorithm also allows optimizing networks with several sources or sinks. To do this, it is necessary to introduce, for example, a fictitious source connected to real sources by arcs, the throughput of which is many times overstated and exceeds the possible flows from real sources.

The graph of the network under consideration is regular, which makes it possible to specify its structure by changing only the parameter N .

The $DrawNetwork(G)$ operator is used to display the graph. Note that the ability to display directed graphs in the Maple system appeared only in its latest versions.

The first chromosome of the initial generation is formed by changing the weights of randomly selected J pairs of arcs. In one arc, the weight decreases by the step value, in the other, it increases by the same amount. Only real arcs are considered, that is, arcs with non-zero weight:

```

step:=0.1:
for m to J do
  k0:=0:
  k1:=nomRand(); k2:= nomRand();
  for u to N do
    for v to N do if (A0[u,v]<>0) then
      k0:=k0+1: if k0=k1 then A1[u,v]:=A0[u,v]+step: fi:
      if k0=k2 then A1[u,v]:=A0[u,v]-step: fi:
    fi:
  end:#v
end:#u
end:#m

```

The random number generator $nomRand = rand(1..ke)$ is used here, where $ke = nops(Edges(G))$ is the number of graph edges or the number of nonzero elements of the matrix $A0$. The J -number determines the number of changed balance pairs. In fact, this is the length of the chromosome. Considering that the numbers $k1$ and $k2$ are random and may coincide, the length of the chromosome can be of different lengths. Here $J = 25$ is taken as an example. The flow value calculated by the Ford - Fulkerson algorithm is given by the $MaxFlow(G, 1, N)$ operator built into Maple, in which the graph G is indicated, the source is vertex 1, and the sink is vertex N .

Here is the result of a genetic algorithm with three selected chromosomes, formed over 45 generations of chromosomes (Fig. 2)

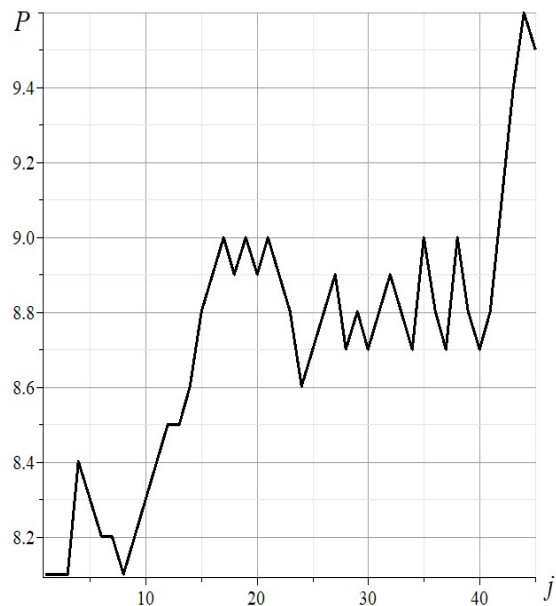


Fig. 2. The value of the stream through the network for 45 generations of the algorithm

As expected, the curve of the dependence of the total flow through the network on the number of iterations (generations of chromosomes) has a chaotic character. Low-amplitude fluctuations alternate with significant jumps, leading the network to an optimal structure. On the one hand, this is due to the presence of a random number operator in the algorithm, on the other hand, such abrupt results are inherent in genetic algorithms. Several numerical experiments carried out with the constructed algorithm have shown that there are several constants in the algorithm that affect the convergence of the result. First of all, this is the step of changing the weights step. For this example, changing the step from 0.09 to 1.5 changes the final flow value by 10-20%. Another parameter is the length of the chromosome. Least of all the result depends on the number of iterations. However, in some cases, with a large number of iterations, the algorithm can get to the descending branch, having passed its maximum.

An image of a graph with changed diameters of arcs is given by the DrawNetwork (G) operator with a previously changed weight matrix $G := \text{Digraph}(A1, \text{weighted})$ (Fig. 3).

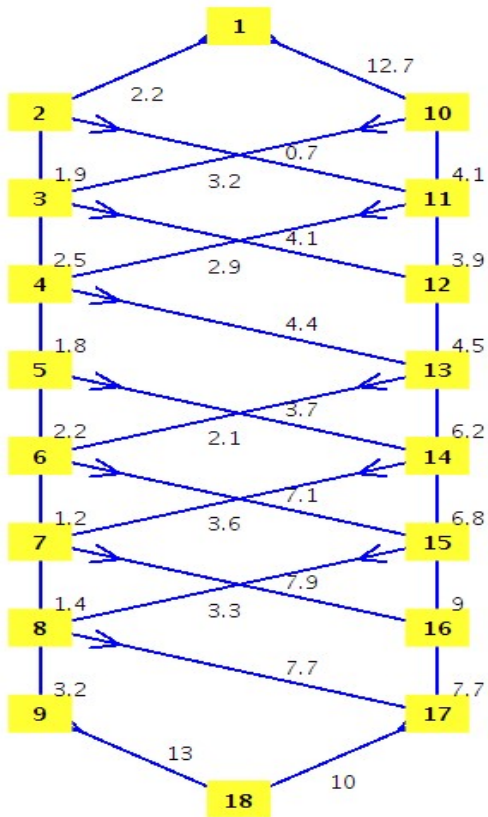


Fig. 3. Optimized network after 45 generations of evolution

Note that, according to the problem statement, the sum of the nominal diameters (throughput) of the original network and the optimized one is the same. Consequently, the new network did not require more material and the throughput increased by almost 25%. At the same time, significant changes have occurred in the network graph scheme. Disappeared, for example, the arc (16,9), since as a result of evolution, its diameter turned out to be equal to zero. This arc does not affect the network bandwidth in any way, and the algorithm transferred its bandwidth to other arcs. It is

also characteristic that the final arcs connected to the drain and having large weights (diameters) did not change their values in the course of evolution.

For comparison, consider a network graph of three parallel branches of order $N = 17$ and size $M = 34$ (Fig. 4). We set the weights of the arcs:

```

k:=5: N:=3*k+2;
for i to k-1 do
  for j to 3 do
    A0[i+k*j-k+1,i+k*j-k+2]:=5+i;
  od;
  for j to 2 do
    A0[i+k*(j-1)+1,i+k*j+2]:=1+j;
    A0[i+k*j+1,i+k*(j-1)+2]:=1+j;
  od;
od;
for i to 3 do
  A0[1,k*(i-1)+2]:=i;
  A0[k*i+1,n]:=5;
od;

```

We choose the total throughput (the sum of the diameters of all arcs) equal to 150, the same as for the previous graph (Fig. 1).

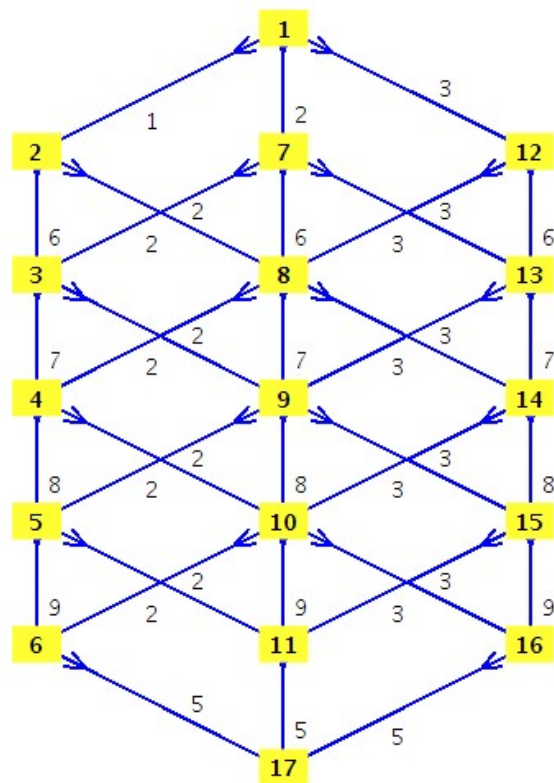


Fig. 4. Planar heat network 2, $N=17$, $M=34$

The step of changing the capacities in this example is chosen equal to $\text{step} = 0.3$, and the length of the chromosomes (the number of pairs of changed arcs) is 32.

The result of the evolutionary process has the same character (Fig. 5), but the flow value is greater than $P = 12.17$. In the process of counting, a false maximum appears on the $j=9$ generation of chromosomes. The modified network is shown in Figure 6. The algorithm redistributed the arc diameters mainly from the drain to the source. The arc (9-5) with a diameter of 0 was actually erased from the diagram.

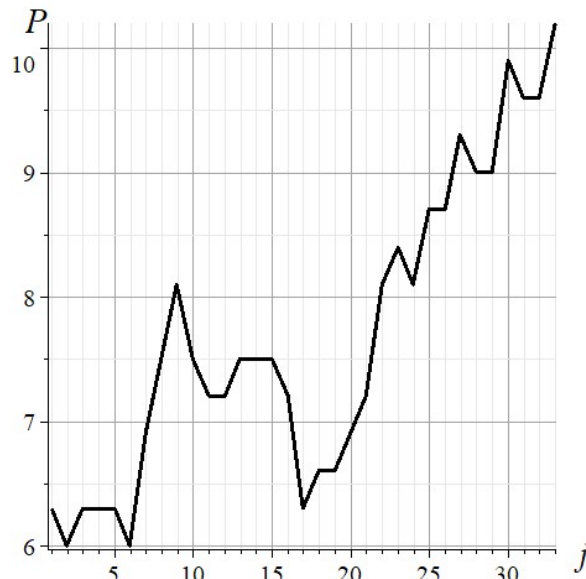


Fig. 5. The value of the stream through the network 2 for 35 generations of the algorithm

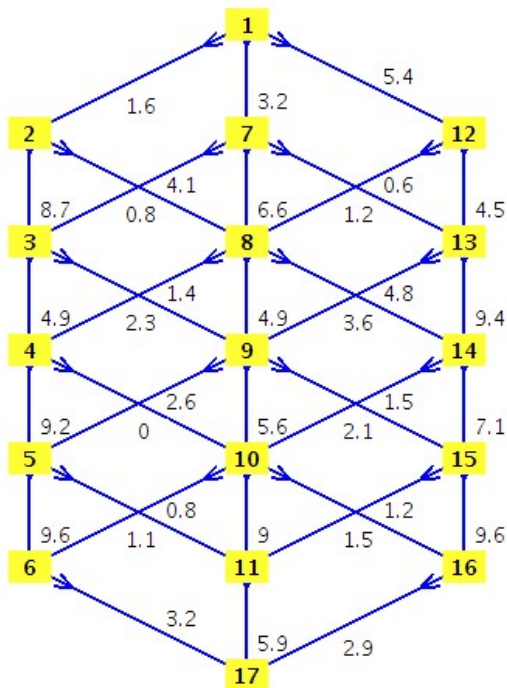


Fig.6. Optimized network after 35 generations of evolution

CONCLUSION

The main task of the algorithm is the problem of network design. For a simple topology of a directed graph with a symmetric structure of arcs, the problem is solved intuitively and is reduced to a uniform distribution of the capacities of arcs. In practice, the networks have already been installed and it is possible to increase the network bandwidth only by redistributing the bandwidth of individual branches. The natural limitation of funds for the construction of the network also implies the condition for limiting the total throughput of all arcs. An algorithm for solving this problem is proposed, based on the known Push-Relabel algorithm built into the Maple system, and the genetic maximum search algorithm. Numerical experiments with the constructed algorithm showed that the result significantly depends on the step of changing the diameters of the arcs and on the length of the chromosome. A similar evolutionary algorithm was previously used in the design of building structures [12].

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