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СВОЙСТВО ВЛОЖЕНИЯ СПЕКТРОВ ЧАСТОТ СОБСТВЕННЫХ КОЛЕБАНИЙ РЕГУЛЯРНЫХ МЕХАНИЧЕСКИХ СИСТЕМ

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Анализируются спектры собственных колебаний некоторых простых регулярных систем. Обнаружено свойство вложения спектров систем с меньшим порядком в спектры систем с большим порядком. Показано, что свойство вложения спектров в классической задаче о колебании упруго соединенных грузов на гладкой плоскости проявляется в зависимости от крепления системы. В задаче о вертикальном колебании равномерно расположенных грузов на упругой невесомой балке получено, что высшая частота не зависит от числа грузов. Определение частот колебаний систем, обладающих свойством вложения, свелось к нахождению собственных чисел бисимметричной матрицы.

Ключевые слова: собственные колебания, свойство вложения, спектр, балка, высшая частота колебаний, бисимметричная матрица

Introduction. Analysis of eigenfrequency spectra of mechanical systems and structures is of great practical importance. In [1] on the specifics of the spectrum of oscillations of the system is proposed to detect defects in products. The temperature and the crystallographic orientation of the lattice, as shown in [2], affect the spectrum of the oscillations of the single-crystal rotor blades of the engine. Analysis of the adequacy of the mathematical model of the compressor-condenser unit in [3] is carried out on the spectrum of oscillations obtained by the finite element method.

Among the mechanical systems and building structures, a class of regular systems can be singled out separately, having periodically repeated elements or groups of elements in their structure [5 -8]. We show that the spectra of some regular systems have one previously unnoticed property — the embedding property of frequency spectra.

Natural oscillations of regular trusses were studied in [9-13].

Cargo system on a smooth plane fixed on one side. Consider the equations of small oscillations of the system of loads connected by linear elastic elements (springs) with stiffness c (Fig. 1).

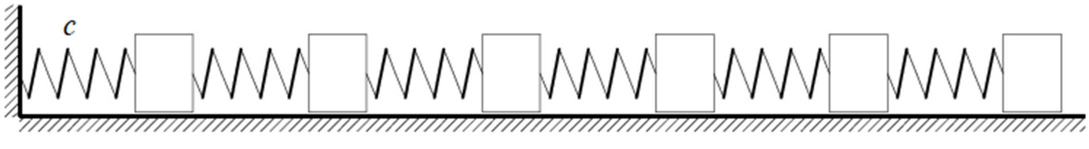

Fig. 1. Cargo system with elastic bonds on a smooth plane, n=6

Applying the Lagrange formalism, we introduce generalized coordinates — linear displacements of loads. The kinetic energy of the system of loads of the same mass m has the form

$$T = m\sum_{k=1}^{n} \dot{x}_k^2 / 2, \tag{1}$$

where $\dot{x}_k$ is the velocity of the *k*-th point. The coordinate x_0 corresponds to the attachment point of the left spring. The potential energy of compression of springs has the form

$$\Pi = c\sum_{k=1}^{n} (x_{k-1} - x_k)^2 / 2, \quad x_0 = 0.$$

We write the system of n Lagrange equations of the 2nd kind (k=1,...,n)

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_k}\right) - \frac{\partial T}{\partial x_k} = -\frac{\partial \Pi}{\partial x_k} \ ,$$

in matrix form

$$m\ddot{\bar{X}} + [D_n]\bar{X} = 0, \tag{2}$$

where $[D_n]$ is the stiffness matrix, $\bar{X}$ — the displacement vector of loads of length n, $\ddot{\bar{X}}$ — vector of accelerations. For n=4 this matrix, for example, has the form

$$[D_4] = \begin{bmatrix} 2c & -c & 0 & 0 \\ -c & 2c & -c & 0 \\ 0 & -c & 2c & -c \\ 0 & 0 & -c & c \end{bmatrix}. \tag{3}$$

Note that this matrix is not symmetric with respect to the side diagonal. If we multiply (2) by the matrix of compliance $[B_n]$ inverse to $[D_n]$, then taking into account the substitution $\bar{X} = \bar{A}\sin(\omega t + \varphi_0)$ equivalent to the replacement $\ddot{\bar{X}} = -\omega^2 \bar{X}$, the problem is reduced to the problem of the eigenvalues of the matrix $[B_n]$: $m\omega^2 [B_n]\bar{X} = \bar{X}$, where $\lambda_k^{(n)}$ the eigenvalue corresponds to the eigenfrequency ω, or $[B_n]\bar{X} = \lambda_k^{(n)}\bar{X}$. At n=4 the matrix $[B_n]$ has the form

$$[B_4] = \frac{1}{c}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}.$$

The elements of the upper (right) triangle of this symmetric matrix in General for an arbitrary value n have the form:

$$b_{i,i+j} = i,\ i = 1,...,n-j,\ \ j = 0,...,n-1.$$

Eigenfrequency spectra for systems with different number of loads at c=1 N/m, m =1 kg are shown by curves (Fig. 2). Conventionally, each curve connects the points corresponding to the frequencies of the system with n masses, and k is the number of frequencies.

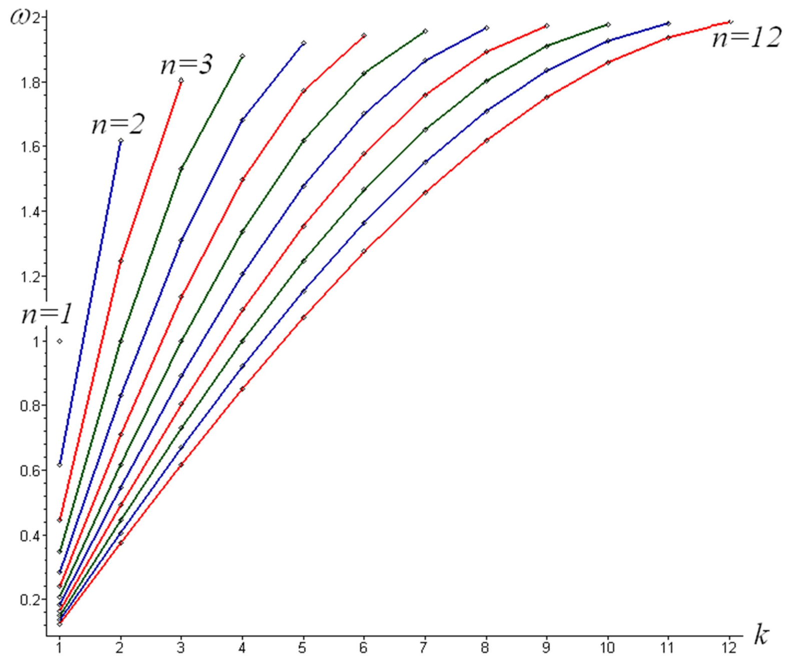

Fig. 2. Eigenfrequency spectra of unilaterally fixed cargoes, c^{-1}

Discovered the next match of frequencies:

$$\omega_1^{(2)} = \omega_2^{(7)} = \omega_3^{(12)} ... = 0,618c^{-1}, \ \omega_2^{(2)} = \omega_5^{(7)} = \omega_8^{(12)} ... = 1,618\,c^{-1},$$
$$\omega_1^{(4)} = \omega_2^{(7)} = \omega_3^{(10)} ... = 1,000c^{-1}, \ \ \omega_2^{(3)} = \omega_5^{(10)} = \omega_8^{(17)} ... = 1,247c^{-1},$$
$$\omega_3^{(3)} = \omega_8^{(10)} = \omega_{13}^{(17)} ... = 1,802c^{-1}, \ ...$$

The observed patterns allow to obtain some frequencies without resorting to calculations:

$$\omega_k^{(5k-3)} = 0,618c^{-1}, \ \omega_{3k-1}^{(5k-3)} = 1,618\,c^{-1}, \ \omega_k^{(3k+1)} = 1,000c^{-1}, \ \omega_{3k-1}^{(7k-4)} = 1,247c^{-1},$$
$$\omega_{5k-2}^{(7k-4)} = 1,802c^{-1}, \ \ k = 1,2,...$$

The list goes on.

Cargo system on the plane, fixed on both sides. The equations of oscillation of loads, connected by springs, not stressed at rest (Fig. 3), almost no different from the previous example. The potential energy of compression of springs has the form:

$$\varPi = c\sum_{k=1}^{n+1}(x_{k-1} - x_k)^2 / 2, \quad x_0 = x_{n+1} = 0.$$

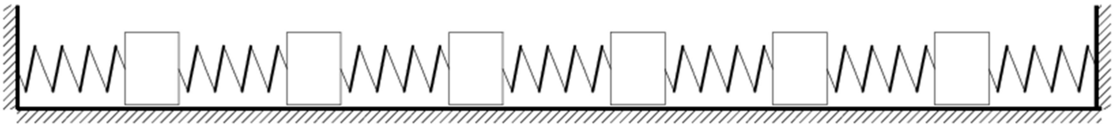

Fig. 3. Cargo system with elastic ties on a smooth plane and double-sided fastening, n=6

The stiffness matrix in (2) at n=4 differs from (3) only in the element on the last line and has the form

$$[D_4]=\begin{bmatrix} 2c & -c & 0 & 0 \\ -c & 2c & -c & 0 \\ 0 & -c & 2c & -c \\ 0 & 0 & -c & 2c \end{bmatrix}.$$

The matrix of compliance inverse to the matrix $[D_n]$ has the property of symmetry. At n=4 it has the form:

$$[B_4]=\frac{1}{5c}\begin{bmatrix} 4 & 3 & 2 & 1 \\ 3 & 6 & 4 & 2 \\ 2 & 4 & 6 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}.$$

A generalization of the form of this matrix to the General case is obvious. The compliance matrices for an arbitrary number of loads n have the form

$$[B_n]=\frac{1}{(n+1)c}\begin{bmatrix} n & n-1 & \dots & 2 & 1 \\ n-1 & 2(n-1) & \dots & 4 & 2 \\ n-2 & 3(n-1) & \dots & 6 & 3 \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 2 & \dots & n-1 & n \end{bmatrix}.$$

Graph of frequency distribution in the spectra (Fig. 4) with a different number of masses is similar to the schedule in figure 2. However, there is a fundamental difference that constitutes the main idea of this article. If in the first problem on the oscillation of loads fixed on the one hand there is only a coincidence of some frequencies, here, in the symmetric problem, the frequency spectra $\{\Omega_k\}, k=1,2,...$ have the property of embedding (Fig. 5): $\{\Omega_\alpha\}\subset\{\Omega_\beta\}$, where α, β is the number of masses in the systems. We have the following relations at k =1,2,…

$$\{\Omega_1\}\subset\{\Omega_{2k+1}\},\ \{\Omega_2\}\subset\{\Omega_{3k+2}\},\ \{\Omega_3\}\subset\{\Omega_{4k+3}\},\ \{\Omega_4\}\subset\{\Omega_{5k+4}\},... \quad (4)$$

Moreover, these relations admit one more simple generalization:

$$\{\Omega_j\}\subset\{\Omega_{(j+1)k+j}\},\ \ j,k=1,2,..., \quad (5)$$

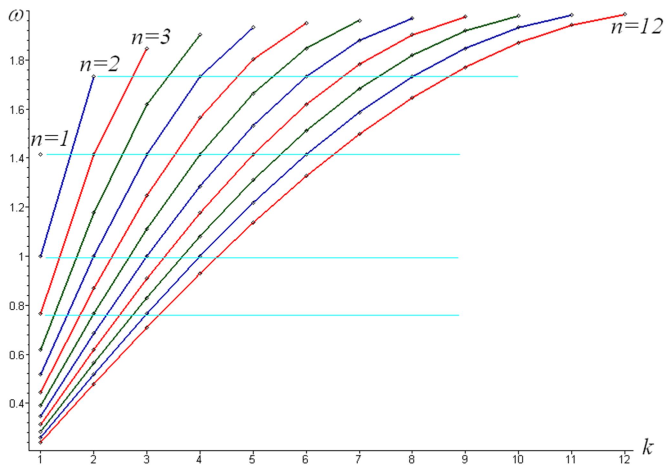

Fig. 4. Spectra of the eigenfrequencies of the system of goods two sides fixed, c^{-1}. Horizontal lines indicate the matching frequency

$n=1$, [1.414]
$n=2$, [1.000, 1.732]
$n=3$, [0.765, 1.414, 1.848]
$n=4$, [0.618, 1.176, 1.618, 1.902]
$n=5$, [0.518, 1.000, 1.414, 1.732, 1.932]
$n=6$, [0.445, 0.868, 1.247, 1.564, 1.802, 1.950]
$n=7$, [0.390, 0.765, 1.111, 1.414, 1.663, 1.848, 1.962]
$n=8$, [0.347, 0.684, 1.000, 1.286, 1.532, 1.732, 1.879, 1.970]
$n=9$, [0.313, 0.618, 0.908, 1.176, 1.414, 1.618, 1.782, 1.902, 1.975]
$n=10$, [0.285, 0.563, 0.831, 1.081, 1.310, 1.511, 1.683, 1.819, 1.919, 1.980]
$n=11$, [0.261, 0.518, 0.765, 1.000, 1.218, 1.414, 1.587, 1.732, 1.848, 1.932, 1.983]
$n=12$, [0.241, 0.479, 0.709, 0.929, 1.136, 1.326, 1.497, 1.646, 1.771, 1.870, 1.942, 1.985]
$n=13$, [0.224, 0.445, 0.661, 0.868, 1.064, 1.247, 1.414, 1.564, 1.693, 1.802, 1.888, 1.950, 1.987]
$n=14$, [0.209, 0.416, 0.618, 0.813, 1.000, 1.176, 1.338, 1.486, 1.618, 1.732, 1.827, 1.902, 1.956, 1.989]

Fig. 5. The attachment of the spectra of eigenfrequencies of the system loads, mounted on two sides

Loads on an elastic beam (version 1). Consider the vertical oscillations of the system of loads of mass *m*, located on a linearly elastic weightless beam at the same distance from each other and from the supports (Fig. 6).

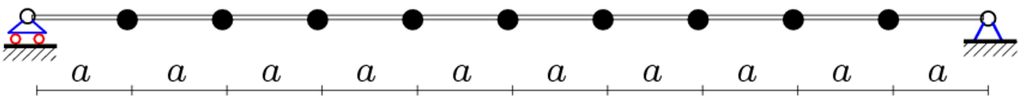

Fig. 6. Beam with masses, n=9 (version 1)

The differential equation of small oscillations of this system does not differ from equation (2), where $\bar{X}$ – the vector of vertical displacements of masses, and the elements of the matrix of compliance, inverse to $[D_n]$, are calculated by the Maxwell-Mohr formula:

$$b_{i,j} = \int_0^L \frac{m_i m_j}{EJ} dx,$$

where $m_i(x)$ and $m_j(x)$ are the diagrams of bending moments in the beam from the action of vertical unit forces applied to the places of masses *i* and *j* (Fig. 7), where *EJ* is the bending rigidity of beams. Diagrams of moments included in the Maxwell – Mohr formula have the form

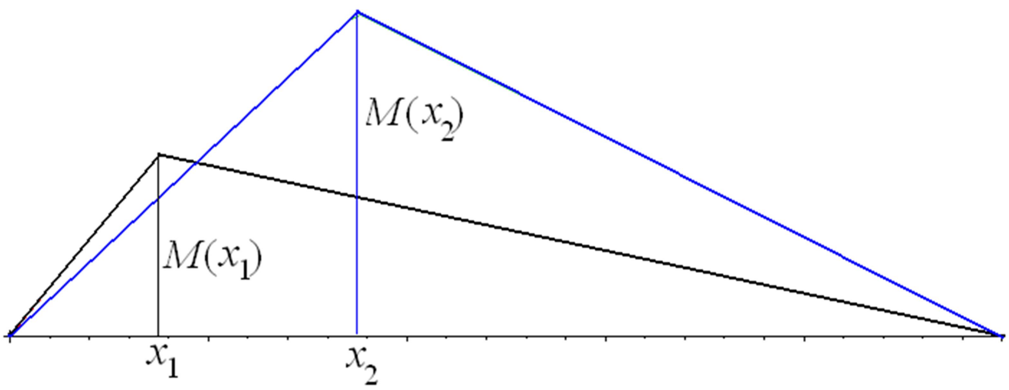

Fig. 7. Moment plots for calculating the elements of the compliance matrix

Maximum plot values:

$$M_k(x_k) = x_k(L - x_k)/L,\ k = i,\ j, \tag{6}$$

where L=(n+1)a is the beam length. If the loads are at the same distance from each other and from the supports (span split with step a), then $x_k = ak,\ k = i,\ j.$ Plots have an analytical expression

$$m_k(x) = \begin{cases} xM_k(x_k)/x_k,\ 0 \le x \le x_k, \\ (L-x)M_k(x_k)/(L-x_k),\ x_k \le x \le L. \end{cases}$$

Thus, the coefficients of the compliance matrix have the form

$$b_{i,j} = \int_0^L \frac{m_i m_j}{EJ}dx = \frac{i(j-n-1)(i^2+j^2-2j(n+1))a^3}{EJ},\ j > i,$$

$$b_{j,i} = b_{i,j},\ j \le i.$$

At n = 4 the matrix has the form:

$$[B_4] = \frac{a^3}{30EJ}\begin{bmatrix} 32 & 45 & 40 & 23 \\ 45 & 72 & 68 & 40 \\ 40 & 68 & 72 & 45 \\ 23 & 40 & 45 & 32 \end{bmatrix}.$$

The eigenvalue $\lambda_k^{(n)}$ of this matrix corresponds to the eigenfrequency $\omega_k^{(n)} = 1/\sqrt{\lambda_k^{(n)}m}$. The spectra of own frequencies of fluctuations, related to $\sqrt{EJ/(ma^3)}$, for beams with various numbers of goods displayed curves (Fig. 8) similar curves in figures 2 and 4.

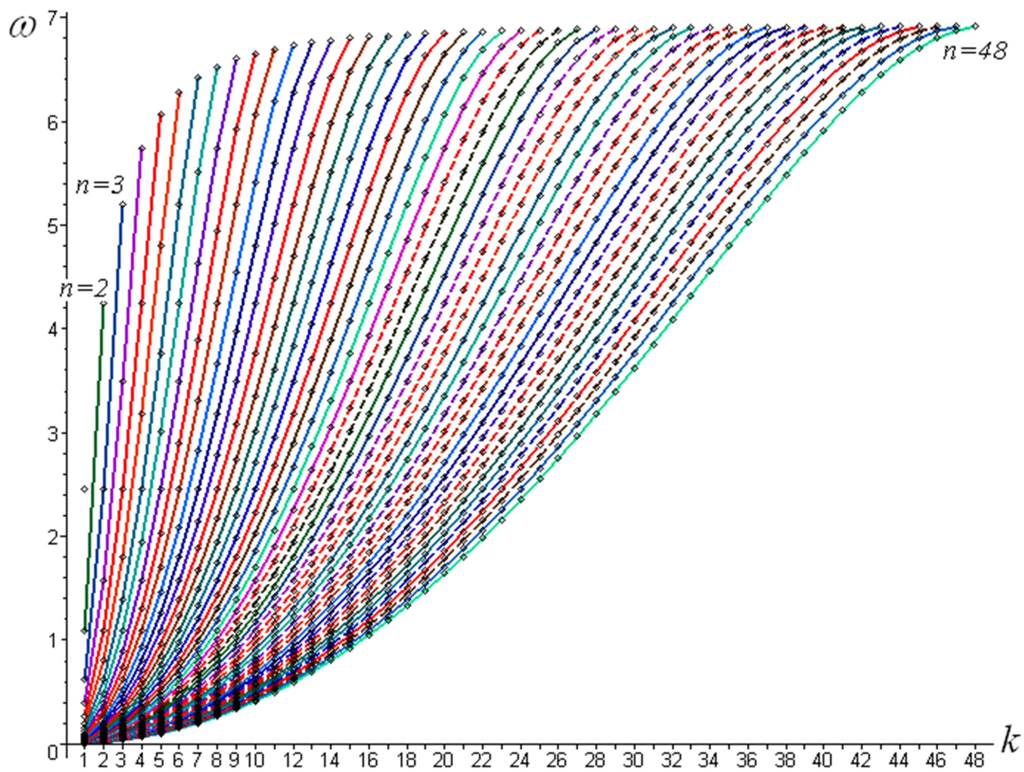

Fig. 8. Natural frequency spectra of loads on the beam, n = 1-48 (version 1)

$n = 1$, [2.449]
$n = 2$, [1.095, 4.243]
$n = 3$, [0.617, 2.449, 5.201]
$n = 4$, [0.395, 1.575, 3.487, 5.742]
$n = 5$, [0.274, 1.095, 2.449, 4.243, 6.070]
$n = 6$, [0.201, 0.805, 1.807, 3.176, 4.793, 6.281]
$n = 7$, [0.154, 0.617, 1.385, 2.449, 3.766, 5.201, 6.424]
$n = 8$, [0.122, 0.487, 1.095, 1.942, 3.008, 4.243, 5.507, 6.525]
$n = 9$, [0.099, 0.395, 0.888, 1.575, 2.449, 3.487, 4.628, 5.742, 6.599]
$n = 10$, [0.082, 0.326, 0.734, 1.303, 2.030, 2.903, 3.895, 4.943, 5.925, 6.654]
$n = 11$, [0.069, 0.274, 0.617, 1.095, 1.708, 2.449, 3.305, 4.243, 5.201, 6.070, 6.697]
$n = 12$, [0.058, 0.234, 0.525, 0.934, 1.457, 2.092, 2.831, 3.658, 4.539, 5.414, 6.187, 6.730]
$n = 13$, [0.050, 0.201, 0.453, 0.805, 1.257, 1.807, 2.449, 3.176, 3.969, 4.793, 5.592, 6.281, 6.757]
$n = 14$, [0.044, 0.175, 0.395, 0.702, 1.095, 1.575, 2.138, 2.779, 3.487, 4.243, 5.012, 5.742, 6.366, 6.779]

Fig. 9. The attachment of the spectra of eigenfrequencies of the system loads on the beam. Same frequency are underlined

There is also an embedding of spectra of the form (4) and (5) found in the problem of loads with elastic bonds on a smooth plane (Fig. 3). The dimensionless oscillation frequency $\omega_1^{(1)} = \sqrt{6} = 2{,}449$ at n=1 coincides with the known result for the oscillation frequency of the load on a beam length $2a$: $\omega = \sqrt{6EJ/(ma^3)}$ [14].

Loads on a shortened elastic beam (version 2). Consider another fastening of masses, also evenly distributed over the beam (Fig. 10). The difference from the previous problem is the shortened distances of the initial and last masses to the supports. Here the beam length L=na.

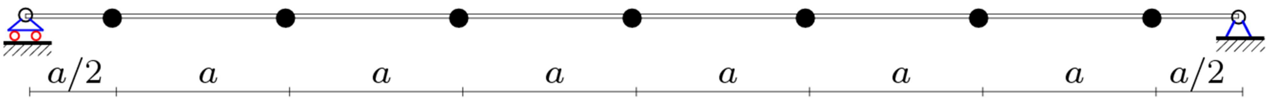

Fig. 10. Beam with masses, n=7, (version 2)

The maximum values of the moments plots are calculated by the same formulas (6), where $x_k = a(k-1/2)$, $k = i, j$. The coefficients of the compliance matrix, the eigenvalues of which determine the oscillation frequency, have the form

$$b_{i,j} = \int_0^L \frac{m_1 m_2}{EJ} ds = a^3(2i-1)(2j-2n-1)(2(i^2+j^2-i-j-2nj+n)+1)/(48nEJ), \ i<j,$$

$$b_{j,i} = b_{i,j}, \ j \le i.$$

At n=4 the matrix has the form

$$[B_4] = \begin{bmatrix} 49 & 95 & 81 & 31 \\ 95 & 225 & 207 & 81 \\ 81 & 207 & 225 & 95 \\ 31 & 81 & 95 & 49 \end{bmatrix}.$$

Let's write the sets of eigenvalues $\Lambda^{(n)} = \{\lambda_1^{(n)}, \lambda_2^{(n)}, \ldots \lambda_n^{(n)}\}$ of several matrices, where the lower index means the eigenvalue number, the upper – the order of the system. Let us refer these values to $\lambda_1^{(1)} / 48 = a^3 / (EJ)$:

$\Lambda^{(1)} = \{1\}$, $\Lambda^{(2)} = \{1, 8\}$, $\Lambda^{(3)} = \{1, 8/3, 40\}$,

$\Lambda^{(4)} = \{1, 8, 64 \pm 44\sqrt{2}\}$, $\Lambda^{(5)} = \{1, 52/5 \pm 4\sqrt{5}, 156 \pm 68\sqrt{5}\}$,

$\Lambda^{(6)} = \{1, 8/3, 8, 40, 320 \pm 184\sqrt{3}\}$.

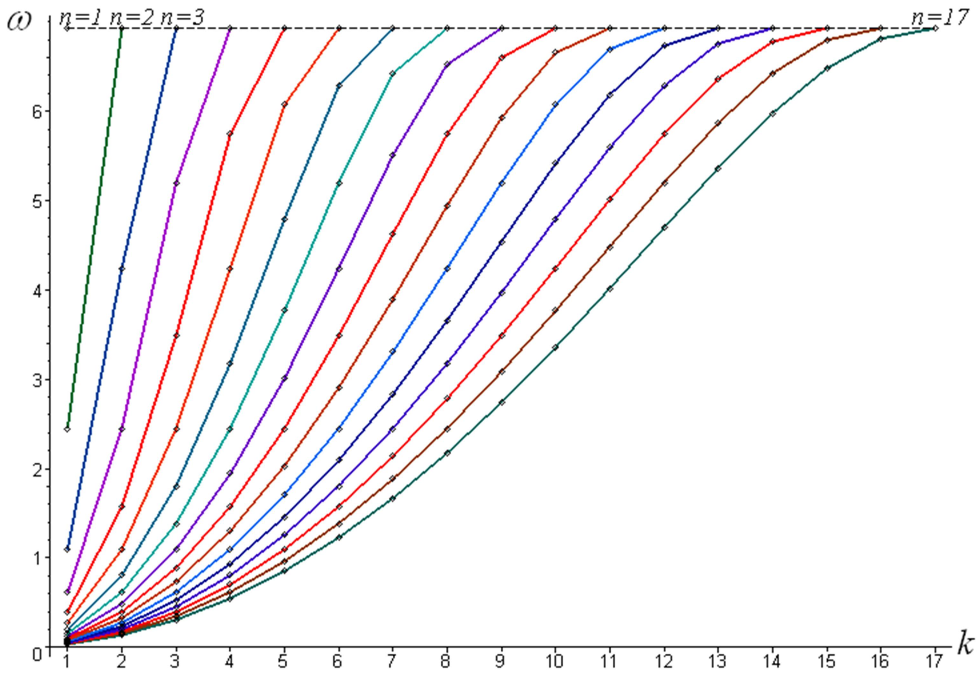

Fig. 11

You can see that the embedding of spectra here is simpler than (4). At k=1,2,… we have:

$$\{\Omega_1\} \subset \{\Omega_k\},\ \{\Omega_2\} \subset \{\Omega_{2k}\},\ \{\Omega_3\} \subset \{\Omega_{3k}\},\ \{\Omega_4\} \subset \{\Omega_{4k}\},$$

or $\{\Omega_j\} \subset \{\Omega_{jk}\},\ j,k = 1,2...$. The latter equality also means that if the number of masses with this arrangement on the beam is expressed as a *Prime* number, then in the case of joint operation of several similar systems with different number of loads, there will be no internal resonance phenomenon. Another property of the frequency spectra found is that the higher frequency here does not depend on the number of loads n, while for the first variant of the cargo arrangement (Fig. 6) this frequency increases with n smoothly, asymptotically approaching the value $\omega = \sqrt{48EJ/(ma^3)} \approx 6{,}928 c^{-1}$ (Fig. 8) — the highest frequency for the second variant of the location of goods. In fact, this corresponds to the principle of Saint-Venant.

Conclusion. The analysis of the spectra of frequencies of natural oscillations of two simple regular systems have identified a property investment or spectra match the frequency of the systems of different order. This property makes it possible in some cases to easily obtain simple solutions to the eigenfrequency problems of large-dimensional systems, reducing them to simple systems. A simple illustrative representation of the frequency spectra in the form of certain curves connecting the frequency points of the spectra is proposed. Formulas for matrix elements whose eigenvalues determine the frequencies of the system are obtained for an arbitrary order of the system. In the problem of small oscillations of loads on the beam it is shown that the higher frequency of oscillations does not depend on the number of loads.

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THE PROPERTY OF EMBEDDING OF NATURAL FREQUENCIES SPECTRA OF REGULAR MECHANICAL SYSTEMS

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The frequency spectra of natural oscillations of some simple regular systems are analyzed. The property of embedding the spectra of systems with a smaller order in the spectra of systems with a large order has been found. It is shown that the property of embedding spectra in the classical problem of oscillation of elastically connected loads on a smooth plane is manifested depending on the mounting system. In the problem of vertical oscillation of uniformly located loads on an elastic weightless beam, it was found that the highest frequency does not depend on the number of loads. Determination of the oscillation frequencies of systems with the property of an embedding is reduced to finding eigenvalues of a bisymmetric matrix.

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