BUILDING MECHANICS

UDC 624.04

M. N. Kirsanov[1]

ANALYTICAL CALCULATION, MARGINAL AND COMPARATIVE ANALYSIS OF A FLAT GIRDER

National Research University «MPEI»
Russia, Moscow, tel.: (495)362-73-14, e-mail: c216@Ya.ru
[1]D. Sc. in Physics and Mathematics, Prof. of Dept. of Theoretical Mechanics and Mechatronics

Statement of the problem. Girder design of statically determinate truss with diagonal lattice unusual species is examined. Using induction, a consistent calculation of truss s with different number of panels displayed the ultimate formula for the deflection of the truss . The result is compared with the analytical expression of the deflection for two standard schemes girder for which expressions of the troughs are also obtained.

Results. Analytical expressions for the deflection of the investigated truss s and two truss s in comparison based on the number of panels, size and load are obtained. Asymptotic and limiting properties of the truss are identified.

Conclusions. It is shown that for a certain number of panels and dimensions of the proposed scheme truss more stringent than the truss with standard bars. Analyzing the stability of compressed rods and not imposing a limit on the strength of the tie bars considering the formula trough, the conclusion is made about the influence of redistribution of the material between the belts on the overall stiffness of the truss .

Keywords: truss , deflection, ultimate properties, analytical solution.

Introduction

A numerical calculation of any structures with numerous members normally comes down to solving a system of linear large dimensional equations. That means an inevitable loss of accuracy which is sometimes unacceptable. Alternatively an analytical calculation is used as modern systems of symbolic mathematics (*Maple, Mathematica, Maxima,* etc.) are very common to use. However, all of these seems rather time-consuming which means there are solutions (even though absolutely accurate for a particular task) obtained only for systems with just a few members.

In [1] the induction method of obtaining solutions for a flat regularly structured truss is described. In [2—4] it is tested for different flat and spatial rod structures. The results obtained using the induction method have no limitations for a number of members and allow for comprehensive formulas for efforts in rods, deflections and frequencies of nodes a truss [5]. The effect of errors in the assemblage on the rigidity and strength of a flat truss in an analytical form is studied in [6], the formula for a deflection of an arched truss is obtained in [7] and that of a flat girder truss with a triangulated lattice in [8]. Analytical approaches are also employed for optimization of the shape of belts of trusses with the help of genetic algorithms [9] and those identical to ant ("bee-like" [10]).

In [11] using the analytical method the influence of the evenness of the number of panels on kinematic changes of a truss. In this paper the induction method is used to obtain and analyze a formula for deflections with a non-standard scheme of a lattice.

1. Scheme of a truss. A truss consists of $2n$ slabs and $m+3$ rods including 3 support rods modeling the support where $m=8n+5$ (Fig. 1). The number of nodes (number of joints (nodes) of a truss is $k=4n+4$, which yields $8n+8$ equations of balance. Therefore the number of rods $m+3=8n+8$ corresponds to the number of static equations, a truss is statistically determinate. The scheme of a truss is a periodic statistically determined system the significance of which is dwelled upon in [12]. Let us look at a load on the lower belt by identical forces *P*.

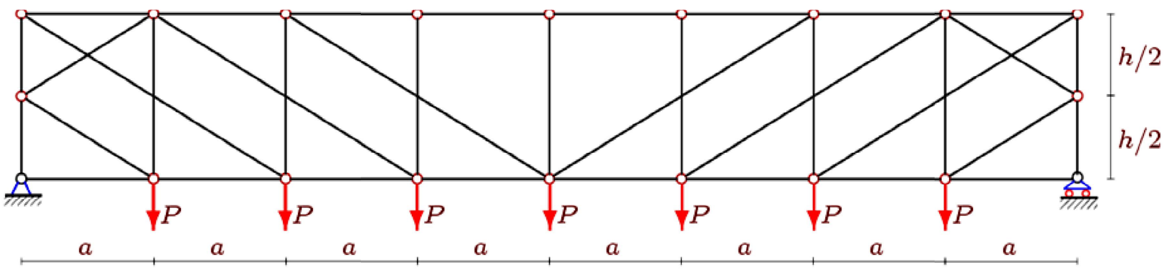

Fig. 1. Truss; n = 4

2. Calculations of efforts. The calculation was performed according to the algorithm [13] in a system of computer mathematics *Maple.* In order to specify the scheme of a structure, it is necessary that joints and order of joining the nodes are introduced into the software. Let us number the joints of a truss (the lower belt is from the left to the right and then the upper belt). Choosing the start of the coordinates in the left motionless joint support we have

$$x_i = x_{i+2n+1} = (i-1)a, \quad y_i = 0, \quad y_{i+2n+1} = h, \quad i = 1,\dots,2n+1,$$

$$x_{4n+3} = 0, \quad y_{4n+3} = y_{4n+4} = h/2, \quad x_{4n+4} = x_{2n+1}, \quad y_{4n+3} = h/2 \ .$$

The order of joining joints of the node lattice is determined by conditional vectors $\overline{V}_i,\ i=1,...,m+3$. The coordinates of these vectors are numbers of joints along the ends. The start and end of the vectors are chosen randomly and in no way are they connected with the signs of the efforts in the rods.

For rods of the lower belt we have the following vectors:

$$\overline{V}_i=[i,i+1],\ i=1,...,2n\,,$$

for the upper belt:

$$\overline{V}_{i+2n}=[i+1+2n,i+2+2n],\ i=1,...,2n\,,$$

for racks of lattices:

$$\overline{V}_{i+4n}=[i+1,i+2n+1],\ \overline{V}_{i+4n}=[i+1,i+2n+2],\ i=1,...,2n-1\,,$$

for braces of lattices:

$$\overline{V}_{i+6n-1}=[i+2,i+2n+1],\ \overline{V}_{i+7n-2}=[i+n,i+3n+3],\ i=1,...,n-1\,,$$

for side racks and braces:

$$\overline{V}_{8n-2}=[2,4n+3],\ \overline{V}_{8n-1}=[2n,4n+4],\ \overline{V}_{8n}=[2n+3,4n+3],\ \overline{V}_{8n+1}=[4\mathrm{n}+4,4n+1],$$

$$\overline{V}_{8n+2}=[1,4n+3],\overline{V}_{8n+3}=[2n+2,4n+3],\ \overline{V}_{8n+4}=[4\mathrm{n}+4,4n+2],\ \overline{V}_{8n+5}=[2n+1,4n+4]\,.$$

The direction cosines which are part of equations of the nodes of a truss are computed using the lengths of rods and projections of their vector presentations on the coordinate axes:

$$l_i=\sqrt{{l_{1,i}}^2+{l_{2,i}}^2}\,,\ l_{1,i}=x_{V_{2,i}}-x_{V_{1,i}},\ l_{2,i}=y_{V_{2,i}}-y_{V_{1,i}},\ \ i=1,...,m_0,$$

where $m_0=m+3$ is a number of rods of a truss including three rods that correspond with a motionless and non-motionless supports. The first index in the number $V_{j,i}$ denotes the number of vector component $\overline{V}_i$, the second one — the number of a rod. The matrix of direction cosines $\mathbf{G}$ has the following members:

$$G_{k,i}=-l_{j,i}\,/\,l_i,\ k=2V_{i,2}-2+j,\ \ k\le m_0,\ j=1,2,\ i=1,...,m_0,$$

$$G_{k,i}=l_{j,i}\,/\,l_i,\ \ k=2V_{i,1}-2+j,\quad k\le m_0,\ \ j=1,2,\ i=1,...,m_0.$$

Identifying the efforts in rods of a truss means solving a system of linear equations which is as follows in the matrix form:

$$\mathbf{G}\overline{S} = \overline{B}. \qquad (1)$$

Here $\overline{S}$ is a vector of unknown efforts containing three supports; $\overline{B}$ is a vector of loads with the length m_0. Horizontal loads applied to the node i are written as part of odd members B_{2i-1}, vertical ones as part of even B_{2i}. The solution of a system of linear equations are identified using a reverse matrix $\overline{S} = \mathbf{G}^{-1}\overline{B}$. This method is well implemented in *Maple* system [14] and requires no purpose-designed linear algebra software package *LinearAlgebra*.

The deflection of a central node of the lower belt of a truss is determined using Maxwell-Mohr formula:

$$\Delta = \sum_{k=1}^{m} \frac{S_k s_k l_k}{EF_k}, \qquad (2)$$

where S_k are efforts in the rods of a truss under an external load; s_k are efforts of a single load applied to the central node in the middle of a span; l_k are lengths of rods. The rod material is assumed to be identical and the modulus of elasticity of all of the rods is E. The area of a section of the lower belt is assumed to be $F_k = 2gF_0,\ k = 1,...,2n$, for the upper belt $F_k = 2(1-g)F_0,\ k = 2n+1,...,4n$, lattices (slanting braces and racks) — $F_k = pF_0$, $k = 4n+1,...,m$. A multiplier $0 < g < 1$ that redistributes the area of a section along the lower and upper belt is chosen so that at $g = 1/2,\ p = 1$ the areas of the sections of all the rods of a truss are identical and are F_0. Let us call the parameter p the coefficient of the reinforcement of the lattice. As g increases, so does the rigidity of the lower belt and that of the upper belt decreases and the total consumption of a material on the belt remains the same.

In order to obtain the formula for deflection, let us use the induction method. Sequentially solving the problem for trusses with 1, 2, 3, etc. panels in the half of the span, let us first of all determine the sequences of whole coefficients proceeding the corresponding expressions and their shared members. We obtain the following

$$\Delta_0(n) = P\frac{a^3(D_n/g + B_n/(1-g)) + (h^3H_n + c^3C_n)/p}{8h^2EF_0}, \qquad (3)$$

where
$$\begin{aligned} B_n &= (27+2n(5n^3+12n^2+4n-36)+(24n-27)(-1)^n)/12, \\ C_n &= ((-1)^n(n-1)+1-n+2n^2)/2, \\ D_n &= (2n^2(4-12n+5n^2)-3((-1)^n-1))/12, \\ H_n &= (3(-1)^n(n-1)+3+n+2n^2)/2. \end{aligned} \tag{4}$$

3. Analysis and comparison. Let us look at a comparison truss 1 (Fig. 2) with the identical sizes, load and sections of the rods constant for the entire truss $F = F_0$.

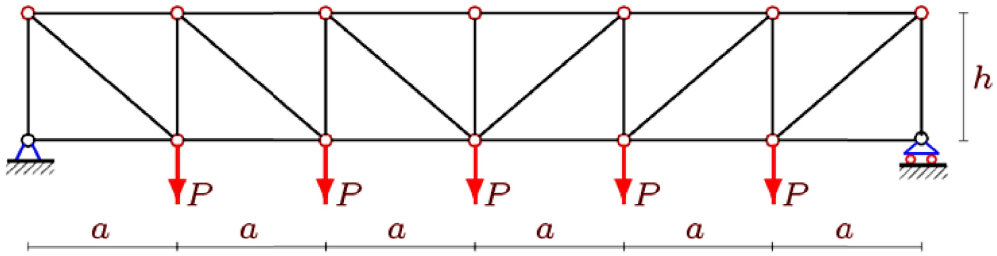

Fig. 2. Comparison truss 1; $n = 3$

The induction method is used to obtain the formula for deflection:

$$\Delta_1(n) = \frac{1}{EF}\sum_{k=1}^{m} S_k s_k l_k = P\frac{A_n a^3 + n^2(c^3+h^3)}{2h^2 EF_0}, \tag{5}$$

where $c = \sqrt{a^2+h^2}$, $m = 8n+1$ and the coefficient $A_n = n^2(1+5n^2)/6$ is a shared member of the sequence 1, 14, 69, 216, 525, 1086, 2009, 3424, 5481, 8350, … fitting in with the recurrence

$$A_n = 5A_{n-1} - 10A_{n-2} + 10A_{n-3} - 5A_{n-4} + A_{n-5}. \tag{6}$$

The recurrence equation is obtained by means of the operator *rgf_findrecur* of *genfunc* package of the system *Maple*. Note that in order to use this operator, the even number of sequence coefficients are required. In this case 10 trusses with the number of slabs in the half of the span from 1 to 10 was to be computed in a symbolic form. The coefficients (4) of the formula (3) are more complex as 14 calculations had to be performed and an eighth order recurrence equation had to be solved to obtain them. For the coefficient D_n, we have, e.g.,

$$D_n = 3D_{n-1} - D_{n-2} - 5D_{n-3} + 5D_{n-4} + D_{n-5} - 3D_{n-6} + D_{n-7}.$$

The solution of the recurrence solution (6), i.e. the expression of the shared member A_n is obtained using a standard operator *rsolve* with the initial data $A_1 = 1$, $A_2 = 14$, $A_3 = 69$, … .

Let us now look at a comparison truss 2 (Fig. 3) with a triangulated lattice. The induction method is obtained for the formula for deflection

$$\Delta_2(n) = \frac{1}{EF_0}\sum_{k=1}^{m} S_k s_k l_k = P\frac{A_n a^3 + n^2 d^3}{8h^2 EF_0}, \tag{7}$$

where $d = \sqrt{a^2 + 4h^2}$, $m = 8n - 1$, $A_n = n^2(10n^2 - 1)/3$. It is of interest that the recurrence equation for the coefficient A_n coincides with (6) with the only difference of the initial values: $A_1 = 3,\ A_2 = 52,\ A_3 = 267,\ldots$. The coefficient at d^3 is easy to deduce and requires no 10 extra analytical calculations of a truss with different numbers of slabs.

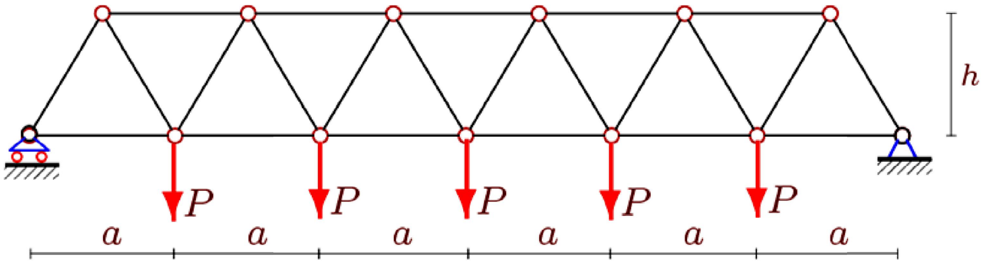

Fig. 3. Comparison truss 2; n = 3

Let us compare the dependencies of the deflection on the number of slabs in three cases. Obviously as the length of a slab a increases, so does the deflection. In order to make the analysis more analytical, let us consider trusses of a constant length and thus $a = L/(2n)$, where L is the length of a span of a truss.

Fig. 4 shows deflections of the truss in question (see Fig. 1) and two comparison trusses (see Fig. 2, 3). The ratio of the rigidity of belts (for the shared constant total area) can be subject to change. Therefore Figure presents two curved lines for this truss — with a thickened lower belt (curved line 3, $g = 0{,}6 > 0{,}5$) and thickened upper belt (curved line 4, $g = 0{,}25 < 0{,}5$).

It is worth noting that for the chosen sizes of the truss these curved lines intersect at $n = 7$. The deflections of the test trusses (comparison trusses 1 and 2) are larger or smaller than the deflection of the truss in question depending on the number of slabs.

Fractures on the curved lines 3 and 4 are due to “flashing” summands in the coefficients (4) including the expression like $(-1)^n$. The curved lines 1 and 2 (almost straight) always approach one another as the number of slabs increases.

The analytical form of presenting the results allows one to obtain a specific ratio of deflections accounting for the behavior of the curved lines in Graph 4. We have the following limit for the comparison truss:

$$\lim_{n\to\infty}\frac{\Delta_1}{\Delta_2}=1.$$

For the truss and comparison truss 2 in question we have

$$\lim_{n\to\infty}\frac{\Delta_0}{\Delta_2}=\frac{1}{4p}.$$

Let us compare the deflection of the truss and comparison truss in question for different reinforcement coefficients of the lattice p. According to (3) and (7), we have the ratio $\Delta_0=\Delta_2$ that can enable us to obtain $p*$ where all the deflections are identical. Let us not fix the length of the span of the truss as was done in designing the graphs in Fig. 4. Hence an increase in n causes an increase in the span. At $g=1/2$ (the belts have identical sections) we have the corresponding expression (equation root $\Delta_0=\Delta_2$):

$$p*=\frac{(2n^2+n+3+3(-1)^n(n-1))h^3+(2n^2-n+1+(-1)^n(n-1))c^3}{2(a^3(5(-1)^n-3n^2-5+4n(3-(-1)^n))+n^2d^3)}. \tag{8}$$

As seen from Fig. 5, this dependence is strongly sensitive to the height of a truss. For smaller heights the reinforcement coefficient is more than 1, for larger ones it is less than 1. The function $p*(n)$ is clearly inhomogeneous conditioned by the summands in (8) contained in $(-1)^n$. As the deflection n increases, the fluctuations of the graph (particularly for higher trusses) attenuate.

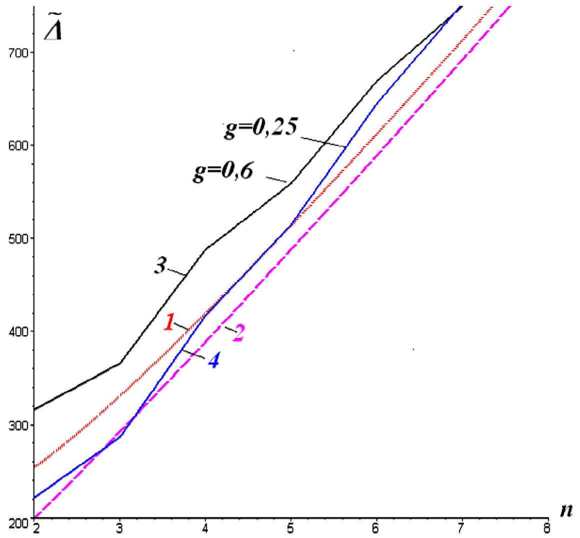

Fig. 4. 1, 2 are comparison trusses 1 and 2, L = 12 m, h = 1 m (see Fig. 2, 3), 3, 4 is a truss (see Fig. 1) at L = 12 m, h = 1 m, p = 1

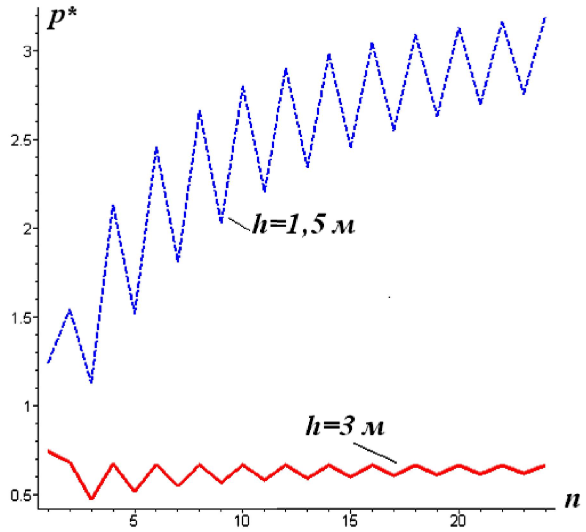

Fig. 5. Coefficient of the reinforcement of the truss $p^*(n)$, determined using the condition $\Delta_0 = \Delta_2$

A specific value of this parameter can be identified

$$\lim_{n\to\infty} p^* = \frac{c^3 + h^3}{d^3 - 3a^3}.$$

Conclusions. The major outcome of the paper is obtaining the formula for the dependence of the deflection on the number of slabs. This has two implications. The first one is bridging "the dimension curse". A drop of accuracy (sometimes a significant drop) hampering the calculations using systems with a large number of rods causes an individual in charge of calculation and design to resort to accurate formulas that are first not always available to use and secondly do not account for all the parameters of a system. All the parameters cannot possibly be accounted for. However in this paper the result (formula) apart from obvious parameters of load, sizes and rigidity also contains coefficients of redistribution of the sections of the areas of the belts g and p. Furthermore, it contains the number of slabs of the truss n. An elementary calculation of a deflection of a truss with a final number of slabs is nothing but an ordinary equation from a university Construction Mechanics course even if the solution is searched for in a symbolic form. Induction generalization of the result on a random number of slabs is not generally possible or is so elaborate that the outcome is unfathomable and impractical. In this task we did get this kind of a solution. The second implication of the formula has to do with testing. For complex and costly computational packages a brief testing ensures high accuracy of calculations.

Here for a girder truss with declining braces where an even load is applied to its lower belt, the formula for the deflection of the middle of the span depending on the size of a truss, num-

ber of spans and load was obtained. The comparison employing analytical formulas for the deflections of trusses with other lattices showed that the ratio of the deflections may vary depending on the size and number of spans. Some specific ratios of the deflections of trusses have been found. A typical inhomogeneity of the dependence of the deflection on the number of slabs is highlighted.

References

1. **Kirsanov, M. N.** Induktivnyj metod resheniya statiki i dinamiki sterzhnevyx sistem / M. N. Kirsanov // Mezhdunarodnyj forum informatizacii. MFI—2001. — S. 163—166.
2. **Kirsanov, M. N.** Izgib, kruchenie i asimptoticheskij analiz prostranstvennoj sterzhnevoj konsoli / M. N. Kirsanov // Inzhenerno-stroitel'nyj zhurnal. — 2014. — № 5 (49). — S. 37—43.
3. **Kirsanov, M. N.** Analiz progiba fermy pryamougol'nogo prostranstvennogo pokrytiya / M. N. Kirsanov // Inzhenerno-stroitel'nyj zhurnal. — 2015. — № 1 (53). — S. 32—38.
4. **Reutov, D. O.** Induktivnyj analiz progiba fermy regulyarnoj struktury v sisteme Maple / D. O. Reutov // Mezhdunarodnaya nauchno-prakticheskaya konferenciya ITON—2014. IV mezhdunar. nauch. seminar i mezhdunar. shkola po matematicheskomu modelirovaniyu v sistemax komp'yuternoj matematiki «KAZCAS—2014»: materialy konf. i tr. seminara. — Kazan': Foliant, 2014. — S. 256—261.
5. **Axmedova, E. R.** Chastotnoe uravnenie dlya ploskoj balochnoj fermy regulyarnoj struktury s treugol'noj reshetkoj / E. R. Axmedova, M. I. Kanatova // Mezhdunar. nauch.-prakt. konf. ITON—2014. IV mezhdunar. nauch. seminar i mezhdunar. shkola po matematicheskomu modelirovaniyu v sistemax komp'yuternoj matematiki «KAZCAS—2014»: materialy konf. i tr. seminara. — Kazan': Foliant, 2014. — S. 198—199.
6. **Kirsanov, M. N.** Induktivnyj analiz vliyaniya pogreshnosti montazha na zhestkost' i prochnost' ploskoj fermy/ M. N. Kirsanov // Inzhenerno-stroitel'nyj zhurnal. — 2012. — № 5 (31). — S. 38—42.
7. **Dzabiev, A. A.** Formuly dlya rascheta progiba arochnoj fermy / A. A. Dzabiev, S. P. Cherepanov // Voprosy obrazovaniya i nauki: teoreticheskij i metodicheskij aspekty: sb. nauch. tr. po materialam Mezhdunar. nauch.-prakt. konf., 31 maya 2014 g. Ch. 4. — Tambov: OOO «Konsaltingovaya kompaniya Yukom», 2014. — S. 63—64.
8. **Zhaketov, D. D.** Progib ploskoj balochnoj fermy s treugol'noj reshetkoj / D. D. Zhaketov, V. B. Yackov // Nauka i obrazovanie v XXI veke: sb. nauch. tr. po materialam Mezhdunar. nauch.-prakt. konf., 31 okt. 2014 g. Ch. 7. — Tambov: OOO «Konsaltingovaya kompaniya Yukom», 2014. — S. 34—36.
9. **Kirsanov, M. N.** Geneticheskij algoritm optimizacii sterzhnevyx sistem / M. N. Kirsanov // Stroitel'naya mexanika i raschet sooruzhenij. — № 2. — 2010. — S. 60—63.
10. **Sonmez, M.** Artificial Bee Colony algorithm for optimization of truss structures / M. Sonmez // Applied Soft Computing. — 2011. — Vol. 11. — P. 2406—2418.
11. **Kirsanov, M. N.** Skrytaya osobennost' i asimptoticheskie svojstva odnoj ploskoj balochnoj fermy / M. N. Kirsanov // Stroitel'naya mexanika i raschet sooruzhenij. — 2014. — № 4. — S. 9—12.
12. **Hutchinson, R. G.** Microarchitectured cellular solids — the hunt for statically determinate periodic trusses / R. G. Hutchinson, N. A. Fleck // ZAMM Z. Angew. Math. Mech. — 2005. — Vol. 85, N 9. — P. 607—617.
13. **Kirsanov, M. N.** Zadachi po teoreticheskoj mexanike s resheniyami v Maple 11 / M. N. Kirsanov. — M.: Fizmatlit, 2010. — 264 s.

14. **Kirsanov, M. N.** Maple i Maplet. Resheniya zadach mexaniki / M. N. Kirsanov. — SPb: Lan', 2012. — 512 c.

15. **Goloshchapov, D.L.** Synthesis of nanocrystalline hydroxyapatite by precipitation using hen's eggshell / D. L. Goloshchapov, V. M. Kashkarov, N. A. Rumyantseva, P. V. Seredin, A. S. Lenshin, B. L. Agapov, E. P. Domashevskaya // Ceramics International. 2013. T. 39. № 4. C. 4539-4549.

16. **Seredin, P.V.** Strukturnye i opticheskie svojstva nizkotemperaturnyh mos-gidridnyh geterostruktur ALGAAS/GAAS (100) na osnove tverdyh rastvorov vychitanija / P. V. Seredin, A. V. Glotov, Je. P. Domashevskja, I. N. Arsent'ev, D.A. Vinokurov, A.L. Stankevich, I.S. Tarasov // Fizika i tehnika poluprovodnikov. 2009. T. 43. № 12. S. 1654-1661.