Models of spatial and planar light bar structures in the Maple system

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Abstract. Models of planar and spatial statically determinate regular trusses and analytical solutions of the problem of calculating natural frequencies are considered. Formulas for frequencies obtained in the Maple system by induction are given. The features of the spectra of regular systems are shown. An algorithm for calculating the estimate of the first frequency by Dunkerley is given. The existence of isolines and spectral constants in the spectra of regular systems is found.

Keywords: truss, vibration frequency, induction, Maple, Dunkerley's estimate

1. Introduction

Most of the engineering problems associated with the analysis of strength, stability, and dynamics of structures are usually solved using numerical methods in well-known software packages based on the finite element method [1]. However, in practice, problems appear in which, due to a large number of structural elements, the accuracy obtained in such solutions turns out to be unsatisfactory. This applies not only to such areas of engineering development as space, transport, aviation, construction of large-span pavements, where any engineering miscalculations associated with inaccurate calculations can have disastrous consequences but also to the precision operation of manipulators of robotic devices. The mounting of microcircuits performed by the robot must have a positioning accuracy greater than the size of the mounted elements. Therefore, in modern calculations, analytical calculations are increasingly used according to formulas, the accuracy of which is determined only by the reliability of the mathematical model of the object under study.

A simple and quite adequate model of a bar structure is a truss with a hinged connection of the bars. The axiom of solidification of statics states that the imposition of constraints on a system of solids in equilibrium does not change its stress states. Consequently, replacing the hinges in the truss with their rigid connection (welding) does not in any way change the forces in the rods, strength, natural vibration frequency, stability, and other operational properties. The simplest hinge scheme mathematically simply models the object, allowing analytical solutions. Here are several well-known models and solutions in the Maple system. The main operators of this system used to obtain solutions are the rgf_findrecur and *rsolve* operators for deriving and solving recurrent equations. The problems of regular statically determinate systems were studied in [2–4]. Analytical solutions of the problem of the deflection of planar trusses were solved in [5–10] using the induction method and the Maple [11,12] symbolic computation system.

2. Method

Modeling the selected scheme of the truss begins with the input into the program of symbolic transformations (Maple, Mathematica, Maxima, etc.) of the coordinates of the nodes and the order of connecting the rods. This is similar to the task of entering a graph in discrete mathematics. Based on these data, a program for calculating the forces in the rods is compiled. As a rule, such programs use the method of cutting nodes [13,14]. The section method is not suitable for all circuits and is inconvenient for programming. It is not required to find the support reactions separately from the forces in the rods. It is quite simple to model the supports with rods, the forces in which will be included in the general system of equilibrium equations. The Maxwell-Mohr formula is used to calculate the spectrum of natural frequencies and to determine the deflection. In the case of calculating the natural frequency using the Dunkerley method [15], for example, the stiffness coefficient is determined by the formula

$$\delta_p = \sum_{j=1}^{K} \left(S_j^{(p)}\right)^2 l_j / (EF).$$

The following symbols are introduced: K – number of elements, $S_j^{(p)}$ – the force in the member with the number j from the action of the vertical unit force applied to the node where the mass with the number p is located, l_j – the length of the rod j. Dunkerley frequency ω_D (lower limit of the first frequency) is calculated through partial frequencies

$$\omega_D^{-2} = m\sum_{p=1}^{K} \delta_p = m\Delta.$$

The main task is to find the dependence of the coefficient Δ on the number of panels. For this, the induction method [16] is used. For this, the Maple system has convenient operators that compose and solve recurrent equations for the coefficients of the sought formula.

3. Results and Discussion

3.1. Scheme 1

In work [17] a truss with a triangular lattice and racks is considered (Fig. 1). The inertial properties of the truss are modeled by the same masses of m nodes of the lower belt. Vibrations of loads are considered to be vertical. The number of degrees of freedom of the cargo system of a truss with n panels is equal to $N = 2n - 1$. The following dependence of the first frequency on the number of panels is obtained

$$\omega_D^{-2} = m((32n^4 + 20n^2 - 7)a^3 + 15(4n^2 - 1)c^3 + 90h^3 n) / (90h^2 EF),$$

where EF is the stiffness of the rods.

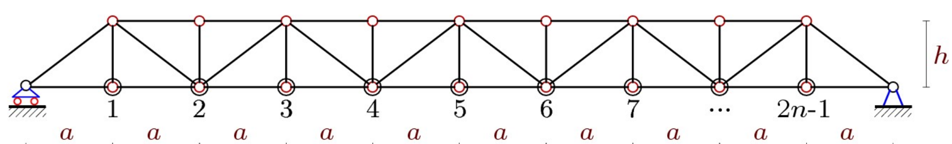

Figure 1. Truss at $n = 6$

It is shown that the accuracy of the solution grows with an increase in the number of panels. The disadvantage of the considered model is the approximation of the inertial properties of the truss only by the masses along the lower belt. More accurate modeling should take into account the lengths of the rods and the distribution of masses across all nodes in proportion to the density of the truss mass associated with the lattice structure.

3.2. Schemes 2 and 3

A truss very similar to truss 1 was considered in [18] (Fig. 2).

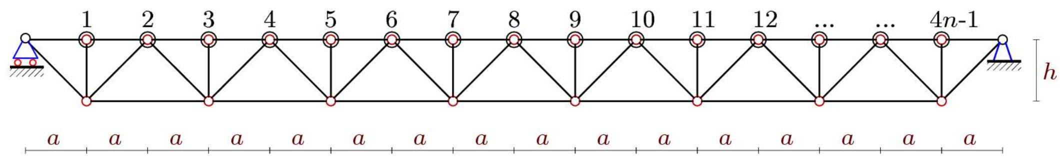

Figure 2. Truss at $n = 4$

The uprights in this truss are not located in all panels, the masses are distributed over the upper chord. By the method of induction on the number of panels, matrices are analyzed that give eigenvalues for determining frequencies. A picture of the frequency spectra of trusses with a different number of panels is obtained. The periodicity of the curves is characteristic (Fig. 3). Also, a frequency can be distinguished on the spectrum, which is the same for trusses of all orders. In ***, when studying the spectrum of the cantilever truss (Fig. 4), this frequency was called the spectral constant. Spectral isolines are also found that asymptotically tend to some constant value (Fig 5).

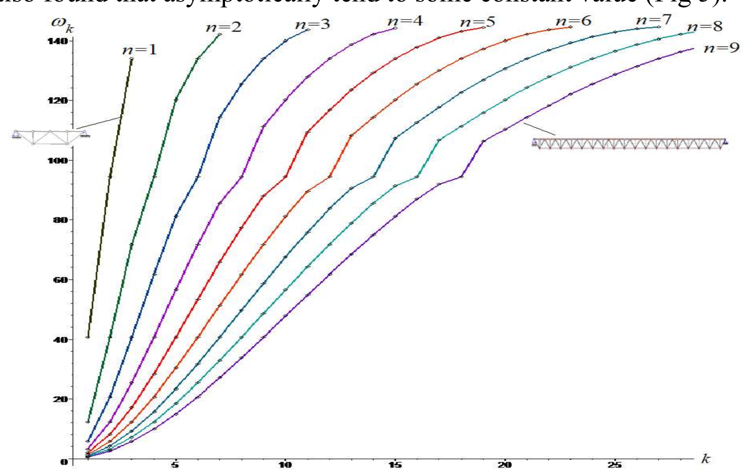

Figure 3. Frequency spectra (rad /s) for trusses with different number of panels

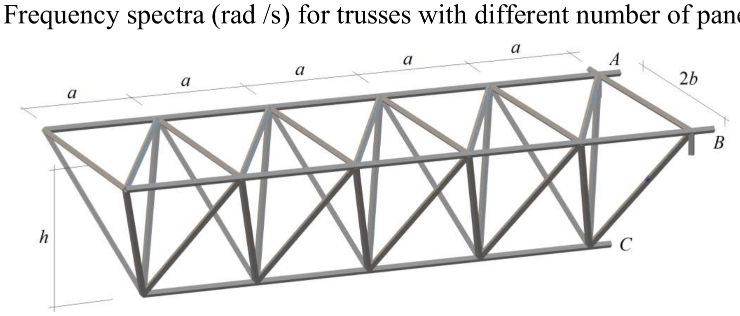

Figure 4. Spatial cantilever truss. Weights are distributed across all nodes

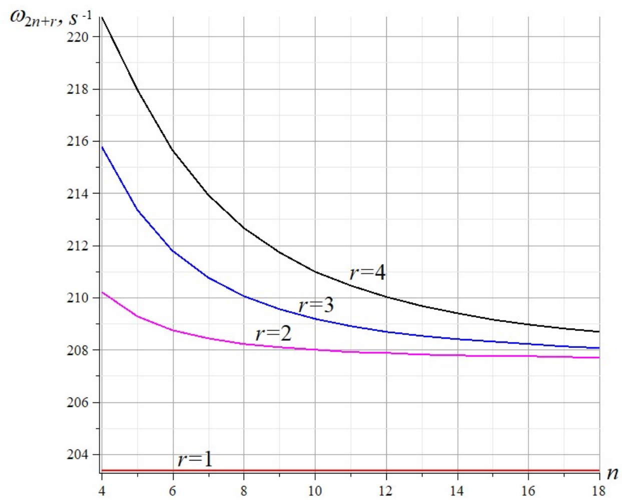

Figure 5. Spatial cantilever truss. Isolines of spectra (r = 2,3,4) and spectral constant (r = 1)

The lower bound according to the Dunkerley method for a spatial truss has the form

$$\omega_D = 4h\sqrt{\frac{EF}{n(6a^3n^3+(32b^3+5c^3+4d^3)n+2a^3+40b^3+4c^3+4d^3)m}},$$

where m is the mass of the load in the node, EF is the stiffness of the rods, $c=\sqrt{a^2+4b^2+4h^2}$, $d=\sqrt{a^2+4b^2}$.

3.3. Scheme 4

The planar cantilever truss, the mass of which is distributed over all nodes (Fig. 6) was studied in [20]. The Dunkerley method yields a lower estimate for the fundamental frequency

$$\omega_D = h\sqrt{EF/(C_1a^3+C_2c^3+C_3h^3)},$$

where $c=\sqrt{a^2+h^2}$, $C_1=n(n-1)(3n^2-n+2)/24$, $C_2=4(8+3n^2+9n^2)$, $C_3=n(3n+1)/4$.

Comparison with the value of the lowest frequency obtained numerically from the solution of the complete spectrum problem showed a very high accuracy of the solution. For $n > 14$, the error does not exceed 5%.

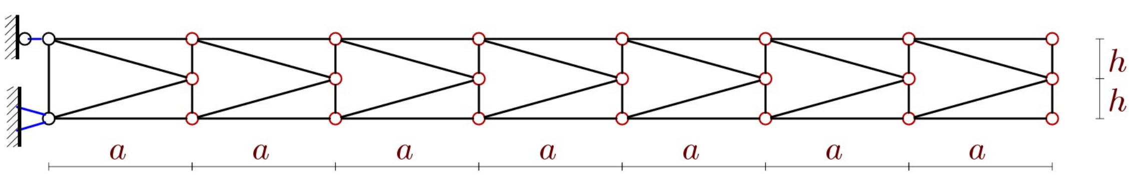

Figure 6. Truss with seven panels

3.4. Scheme 5

In [21] the frequency spectra of a truss with half-span panels with additional horizontal support are considered (Fig. 5). The masses are located at the nodes of the lower belt. The number of degrees of freedom is $2n - 1$. The system of equations for the movement of loads is written in matrix form $[M_n]\ddot{\vec{Y}}+[D_n]\vec{Y}=0$. The solution to the problem of the spectrum of natural frequencies is reduced to finding the eigenvalues of the matrix $[B_n]$ inverse to the stiffness matrix $[D_n]$.

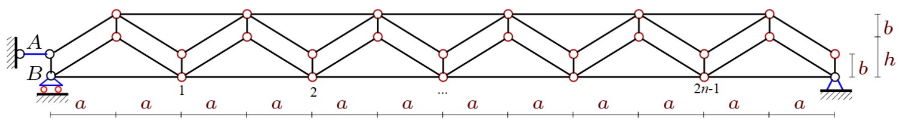

Figure 7. Truss, $n = 3$

The general form of the characteristic matrix is obtained by induction. For $n = 2$ for the matrix

$$[B_2] = \frac{\eta}{n}\begin{bmatrix} 75 & 90 & 57 \\ 90 & 132 & 90 \\ 57 & 90 & 75 \end{bmatrix}$$

the eigenvalues are:

$$\lambda_1 = 9\eta,\ \lambda_{2,3} = 3(22 \pm 15\sqrt{2})\eta.$$

It is indicated here $\eta = \left(a^3 + 2bh^2 + c^3\right)/\left(3h^2EF\right)$. It is possible to obtain eigenvalues for a system of order 3 (five degrees of freedom):

$$\lambda_1 = 9\eta,\ \lambda_2 = 42\eta,\ \lambda_3 = 10\eta/3,\ \lambda_{4,5} = 6(54 \pm 31\sqrt{3})\eta.$$

In the case of a system of order 4 (seven weights at the nodes of the lower belt, seven degrees of freedom), we have analytical expressions

$$\lambda_1 = 9\eta,\ \lambda_{2,3} = 3(22 \pm 15\sqrt{2})\eta,$$

$$\lambda_{4,5} = 3\left(172 \pm 118\sqrt{2} \pm \sqrt{57236 \pm 4062\sqrt{2}}\right)\eta,$$

$$\lambda_{6,7} = 3\left(172 \pm 118\sqrt{2} \mp \sqrt{57236 \pm 4062\sqrt{2}}\right)\eta.$$

The most important result of this work is the observed property of spectra embedding. According to it, the spectra of regular systems of orders n and k are embedded in the spectrum of a system of order nk.

3.5. Scheme 6

Trusses without a bottom chord (Fig. 8) are often used in constructions of larger spans. One of the variants of such a truss was studied in [22]. This is the simplest scheme of this type. An analytical estimate of the lower frequency is obtained by the Dunkerley method

$$\omega_D^{-2} = m\sum_{i=1}^{N}\delta_i = m(C_1a^3 + C_2c^3 + C_3b^3)/(nh^2EF),$$

where $c = \sqrt{a^2 + b^2}$, $C_1 = 16n^2 - 18n + 5,\ C_2 = 16n^2 - 12n + 2, C_3 = 8n^2 - 6n + 1.$

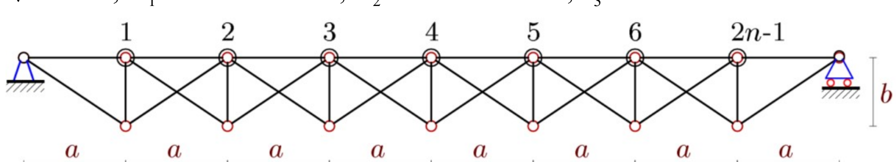

Figure 8. Fink's truss, $n = 4$

4. Conclusion

An overview of some analytical solutions to the problem of determining the natural frequencies of plane statically determinate trusses is given. It is shown that statically determinate regular trusses admit simple analytical solutions for the first frequency and some regular structure of the spectra.

5. Acknowledgments

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