ISSN 1052-6188, Journal of Machinery Manufacture and Reliability, 2017, Vol. 46, No. 1, pp. 46–52. © Allerton Press, Inc., 2017.
Published in Russian in Problemy Mashinostroeniya i Nadezhnosti Mashin, 2017, No. 1, pp. 55–62.

## RELIABILITY, STRENGTH, WEAR RESISTANCE OF MACHINES AND CONSTRUCTIONS

# Design Optimization of Truss Bridge Structures of Composite Materials[1]

**D. V. Tinkov[a] and A. A. Safonov[b]***

[a] *National Research University MPEI (Moscow Power Engineering Institute), Moscow, Russia*
[b] *Skolkovo Institute of Science and Technology, Moscow, Russia*
**e-mail: a.safonov@skoltech.ru*

Received September 7, 2016

**Abstract**– In this paper the problem of optimizing a distribution of carbon fiber content in hybrid glass/carbon fiber reinforced (GFRP/CFRP) structural elements of plane, statically determinate regular elastic truss structure is discussed. A modification of genetic algorithm combined with the method of mathematical induction is proposed. A criterion of structural materials cost minimization while satisfying elastic strength requirements is used as an optimality criterion.

**DOI:** 10.3103/S1052618817010149

Today, structural glass-fiber reinforced plastic (GFRP) components of complex shapes are widely used in aerospace, construction and other industries. Examples of such components include structural elements of composite bridge structures (girders, channels, bridge decking) [1], GFRP elements of power line supporting structures, insulators, elements of aircraft structures, etc.

Being rather expensive, compared to structures made of traditional structural materials such as wood, concrete and metals, polymer composite structures can, nevertheless, successfully compete with their traditional counterparts in many applications [2], especially where a weight or corrosion resistance of a structure is a decisive factor. However, they also have to remain cost effective. One way of achieving this goal is through the optimized use of various reinforcing materials. For example, a development in design solutions for bridge structures is hampered by a relatively low elastic modulus of glass-fiber reinforced plastics. The application of carbon fibers may alleviate these limitations [3, 4].

The goal of this research is to find computational methods to optimize the distribution of CFRP content in truss bridge structures of composite materials. In order to solve the problem a modification of a genetic algorithm [5, 6] has been developed combined with the method of mathematical induction [7]. A criterion of structural materials cost minimization while satisfying stiffness requirements is used as an optimality criterion.

## COMPOSITE BRIDGE STRUCTURES

Today, pedestrian bridges of glass-fiber reinforced plastics are finding ever-increasing application. For example, over 25 bridge structures of glass-fiber reinforced (GFRP) plastics have been designed and manufactured by ApATeCh specialists [1]. Figure 1 shows a general view of the pedestrian bridge (span length 33.37 m) constructed near Pyra settlement at 378-th km of M7 "Volga" highway.

The bridge structure is a spatial joint of four plane trusses with walkway supported by the lower chord. Elements of span, railings (except for mounting groups, joining elements and fixtures) are made of pultruded structural profiles. GFRP profiles are connected with the use of gusset plates and bolts.

Consider a statically determinate elastic regular truss frame (see Fig. 2), loaded at the lower chord. In order to simplify calculations of the deflection a conservative approach is applied: a hinged connection of elements is used in the mathematical model. Following designations are used: $L$ is length of span, $H$ is height of truss, $n$ is number of panels in a half-span, $a$ is length of panels. Figure 2 shows a particular case of the model with four panels in a half-span, $n = 4$.

[1] The article was translated by the authors.

**Fig. 1.**

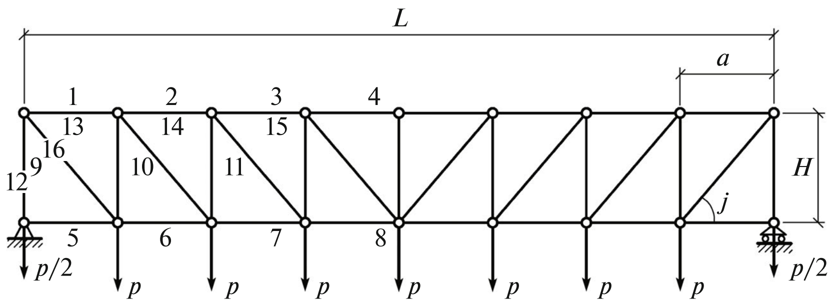


**Fig. 2.**

A continuous numbering of truss bars is used throughout this paper. Due to symmetry conditions, only a half of truss will be considered. Top chord elements are numbered from 1 to $n$; lower chord elements $n + 1, ..., 2n$; vertical posts $2n + 1, ..., 3n$; braces $3n + 1, ..., 4n$.

The weight of truss bars and the weight of a structure are calculated as follows:

$$m_{\mathrm{GF}} = 2\rho_{\mathrm{GF}}\sum_{i=1}^{4n}(1 - k_i)A_i\ell_i, \quad m_{\mathrm{CF}} = 2\rho_{\mathrm{CF}}\sum_{i=1}^{4n}k_iA_i\ell_i, \quad m = m_{\mathrm{CF}} + m_{\mathrm{CF}}, \tag{1}$$

where $A_i$ are the cross section area of structural elements, $\ell_i$ is the length of elements (to the intersection of axes), $\rho_{\mathrm{GF}}$, $\rho_{\mathrm{CF}}$ are the densities of GFRP and CFRP plastics, respectively, $k_i$ is the CFRP content within the volume of a structural element.

Cost of truss bars material can be calculated as follows:

$$c = c_{\mathrm{GF}}m_{\mathrm{GF}} + c_{\mathrm{CF}}m_{\mathrm{CF}}. \tag{2}$$

The central post is not considered in material weight and cost calculations as the force acting on it is zero and its cross section parameters and material characteristics do not influence the total deflection.

The deflection of truss structure can be expressed by Maxwell–Mohr equation [9] as follows:

$$\Delta = 2\sum_{i=1}^{4n}\frac{F_iF_i^{(1)}\ell_i}{\mu_iE_{\mathrm{GF}}A_i}, \tag{3}$$

where $F_i$ is the force acting on a truss bar at design load; $F_i^{(1)}$ is the force acting on vertical posts at unit load applied at midspan in the direction of calculated deflection; $\ell_i$ is the length of a truss bar; $\mu_i$ is the elastic modulus increase factor. Allowable deflection in engineering structures is regulated by relevant construction codes and regulations (e.g., for pedestrian bridges deflection shall not exceed $L/400$ [10]). It is assumed that, for a given span length, the deflection of GFRP truss does not meet stiffness criteria, whereas CFRP truss offer excessive stiffness, therefore, in order to solve the problem an optimal distribution of CFRP within truss members should be determined.

Assuming that the increase in elastic modulus is directly proportional to the volume fraction of CFRP, the relation between coefficients $k_i$ and $\mu_i$ can be expressed as follows:

$$E_{\text{GF}} + k_i\left(E_{\text{CF}} - E_{\text{GF}}\right) = \mu_i E_{\text{GF}}. \tag{4}$$

And $k_i$ can be expressed as:

$$k_i = E_{\text{GF}}\left(\mu_i - 1\right)/\left(E_{\text{CF}} - E_{\text{GF}}\right). \tag{5}$$

Forces acting on truss bars can be determined using the method of joint isolation. In a particular case of a single panel in a half-span ($n = 1$) forces acting on truss bars are calculated as:

$$F_1 = -P/(2\tan\varphi), \quad F_2 = 0, \quad F_3 = -P/2, \quad F_4 = P/(2\sin\varphi).$$

For $n = 2$

$$F_1 = -3P/(2\tan\varphi), F_2 = -2P/\tan\varphi, \quad F_3 = 0, \quad F_4 = 3P/(2\tan\varphi),$$
$$F_5 = -3P/2, \quad F_6 = -P/2, \quad F_7 = 3P/(2\sin\varphi), \quad F_8 = P/(2\sin\varphi).$$

For $n = 3$

$$F_1 = -5P/(2\tan\varphi), \quad F_2 = -4P/\tan\varphi, \quad F_3 = -9P/(2\tan\varphi),$$
$$F_4 = 0, \quad F_5 = 5P/(2\tan\varphi), \quad F_6 = 4P/\tan\varphi,$$
$$F_7 = -5P/2, \quad F_8 = -3P/2, \quad F_9 = -P/2,$$
$$F_{10} = 5P/(2\sin\varphi), \quad F_{11} = 3P/(2\sin\varphi), \quad F_{12} = P/(2\sin\varphi).$$

For purposes of generalization, we calculate forces acting on a truss with arbitrary number of panels, $n$, in a half-span using the method of mathematical induction:

$$\begin{gathered} i = 1, \ldots, n\text{:} \quad F_i = -Pi\left(n - i/2\right)/\tan\varphi; \quad i = (n+1), \ldots, 2n\text{:} \quad F_i = P\left(i - n - 1\right)\left(3n + 1 - i\right)/\tan\varphi; \\ i = (2n+1), \ldots, 3n\text{:} \quad F_i = -P\left(3n + 1/2 - i\right); \quad i = (3n+1), \ldots, 4n\text{:} \quad F_i = P\left(4n + 1 - i\right)/\sin\varphi. \end{gathered} \tag{6}$$

Forces acting on truss elements under the unit load are calculated similarly:

$$\begin{gathered} i = 1, \ldots, n\text{:} \quad F_i^{(1)} = -i/(2\tan\varphi); \quad i = (n+1) \ldots, 2n\text{:} \quad F_i^{(1)} = (i - n - 1)/(2\tan\varphi); \\ i = (2n+1) \ldots, 3n\text{:} \quad F_i^{(1)} = -1/2; \quad i = (3n+1) \ldots, 4n\text{:} \quad F_i^{(1)} = 1/(2\sin\varphi). \end{gathered} \tag{7}$$

It should be noted that forces in equations (6) and (7) are positive for lower chord elements and braces, while for upper chord elements and posts forces are negative irrespective of $n$ and of panel position in a truss.

Expressing the deflection and costs in terms of elastic modulus increase factor $\mu_i$, the deflection equation will feature the modulus increase factor in the denominator, whereas the cost equation will feature modulus increase factor in the numerator, therefore the deflection will reach its limits in case of the lowest cost truss.

## OPTIMIZATION SCHEME

In spite of increasing computational capabilities the complete enumeration of a subset of viable solutions is very cumbersome, thus a genetic algorithm is used to find an optimal solution, based on random and combined enumeration of factors.

We use the standard terminology adopted in genetic algorithms. Varied parameters with specific values are combined into vectors—chromosomes. Chromosomes are formed from modulus increase factors $X\left[\mu_1, \mu_2, \mu_3 \ldots \mu_h\right]$; where $h = 4n$ is the length of a chromosome. Each element of such vector, the gene, corresponds to a specific parameter of a system. Several chromosomes form a population of a next generation and unfit chromosomes are discarded. In the problem considered here the fitness of chromosomes corresponds to the cost of structural elements. The lesser the cost the higher the chromosome fitness is. The number of chromosomes depends on the length $h$ of a chromosome as follows: $m = 2h + 1$. Values of genes of the first population are selected stochastically using random numbers generator. Gene values of next populations are formed by crossover and mutation, based on the three fittest chromosomes. While

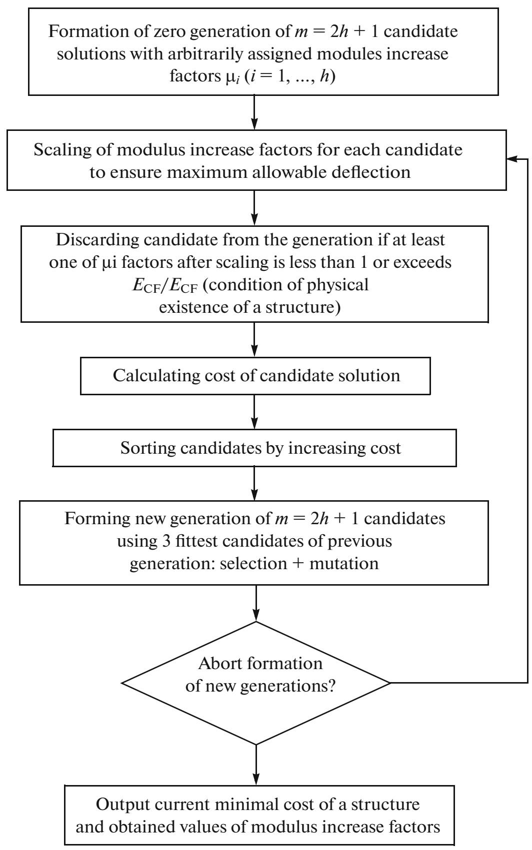


**Fig. 3.**

forming each chromosome a scaling of genes is performed to ensure a maximum allowable deflection for a corresponding chromosome. If, during chromosome formation process, genes appear with values out of the range of $[1;\ E_{\rm CF}/E_{\rm GF}]$, the chromosome is discarded.

Figure 3 shows the procedure of searching for optimal solution based on genetic algorithm.

Table 1 shows an example of forming a new generation from chromosomes of 5 genes long, consisting of 11 chromosomes in one generation. For the generation to be formed a layout of genes from 3 fittest

**Table 1**

Current generatio

| Chromosome no. | Gene no. | | | | | |
|---|---|---|---|---|---|---|
| *m*, *h* | 1 | 2 | 3 | 4 | 5 | |
| 1 | 11 | 12 | 13 | 14 | 15 | |
| 2 | 21 | 22 | 23 | 24 | 25 | Fittest 2 |
| 3 | 31 | 32 | 33 | 34 | 35 | |
| 4 | 41 | 42 | 43 | 44 | 45 | |
| 5 | **51** | **52** | **53** | **54** | **55** | Fittest 1 |
| 6 | 61 | 62 | 63 | 64 | 65 | |
| 7 | 71 | 72 | 73 | 74 | 75 | |
| 8 | 81 | 82 | 83 | 84 | 85 | Fittest 3 |
| 9 | 91 | 92 | 93 | 94 | 95 | |
| 10 | ... | ... | ... | ... | ... | |
| 11 | ... | ... | ... | ... | ... | |

Generation to be formed

| Chromosome no. | Gene no. | | | | |
|---|---|---|---|---|---|
| *m*, *h* | 1 | 2 | 3 | 4 | 5 |
| 1 | **51** | **52** | **53** | **54** | **55** |
| 2 | **51** | **52** | **53** | **54** | 25 |
| 3 | **51** | **52** | **53** | 24 | 25 |
| 4 | **51** | **52** | 23 | 24 | 25 |
| 5 | **51** | 22 | 23 | 24 | 25 |
| 6 | **51** | **52** | **53** | **54** | 85 |
| 7 | **51** | **52** | **83** | 84 | 85 |
| 8 | **51** | **52** | 83 | 84 | 85 |
| 9 | **51** | 82 | 83 | 84 | 85 |
| 10 | **51** | **52** | **53** | X | **55** |
| 11 | Z | **52** | **53** | **54** | **55** |

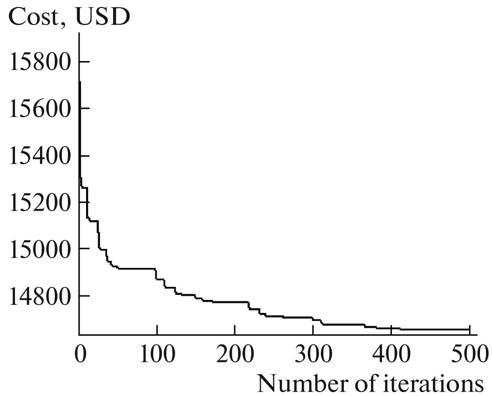


**Fig. 4.**

chromosomes of previous generation is shown. Chromosomes 1–9 are formed by recombination of 3 fittest chromosomes. Chromosomes 10, 11 are formed by mutation of several genes from the fittest chromosome (gene $x$, $z$).

## STUDY OF OPTIMIZATION RESULTS

Consider a structure with span length $L = 36$ m, and height $H = 3.2$ m, having $n = 12$ panels in a half-span (distance between posts $a = 1.5$ m, brace length $d = 3.53$ m). Cross section of upper and lower chords represents a pultruded double channel profile with dimensions 400 × 120 × 18 ($A = 21744$ mm$^2$). Posts and braces are made of pultruded square tube 130 × 10 ($A = 4800$ mm$^2$). Based on data in [1]: GFRP density $\rho_{GF} = 2050\ \text{kg}/\text{m}^3$, CFRP density $\rho_{CF} = 1560\ \text{kg}/\text{m}^3$, cost of GFRP per 1 kg $c_{GF} = 2.62\ USD$, cost of CFRP per 1 kg $c_{CF} = 9.42\ USD$, elastic modulus of GFRP $E_{GF} = 2044\ \text{kgf}/\text{mm}^2$, elastic modulus of CFRP $E_{CF} = 4470\ \text{kgf}/\text{mm}^2$.

Lower chord nodes are loaded with design load of $400\ \text{kgf}/\text{m}^2$ from pedestrian traffic [10]. With walkway width of 3 m a single node experiences force of $P = 900\ \text{kgf}$. The deflection of all-GFRP truss, calculated using the equation (3), constitutes ($\mu_i = 0$): $\Delta = 107.4\ \text{mm}$. The deflection of all-CFRP truss constitutes ($\mu_i = E_{CF}/E_{GF} = 2.19$) $\Delta = 49.1\ \text{mm}$. Allowable deflection [10] constitutes $\Delta^* = L/400 = 90.0\ \text{mm}$. Therefore, GFRP truss does not meet stiffness requirements while CFRP truss far exceeds those requirements.

To compare with the calculated optimal variant, four accurate solutions are considered additionally:

1. Elastic moduli of all elements are increased to levels meeting stiffness requirements ($\mu_i = \tilde{\mu}$).

2. Upper and lower chord moduli are increased ($\mu_i = \tilde{\mu}, \quad i = 1, \ldots, 2n$), while moduli of GFRP posts and braces are left unchanged ($\mu_i = 1, \quad i = 2n + 1, \ldots, 4n$).

3. Upper and lower chord moduli are increased by a factor of $\tilde{\mu}_1$, and moduli of posts and braces are increased by a factor of $\tilde{\mu}_2$.

4. For ease of transportation trusses are assembled of three sections. Eight elements of upper and lower chords are considered made of GFRP. Moduli of central section chords are increased by the factor $\tilde{\mu}_1$. Moduli of 8 posts and braces are increased by the factor $\tilde{\mu}_2$, while moduli of GFRP posts and braces of the central section are left unchanged.

$$\mu_i = 1, \quad i = 1, \ldots, 8; \quad \mu_i = \tilde{\mu}_1, \quad i = 9, \ldots, 12 - \text{upper chord};$$
$$\mu_i = 1, \quad i = 13, \ldots, 20; \quad \mu_i = \tilde{\mu}_1, \quad i = 21, \ldots, 24 - \text{lower chord};$$
$$\mu_i = \tilde{\mu}_2, \quad i = 25, \ldots, 32; \quad \mu_i = 1, \quad i = 33, \ldots, 36 - \text{posts};$$
$$\mu_i = \tilde{\mu}_2, \quad i = 37, \ldots, 44; \quad \mu_i = 1, \quad i = 45, \ldots, 48 - \text{braces}.$$

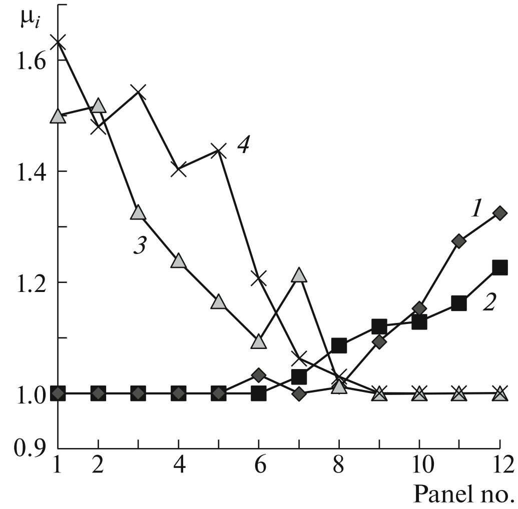


**Fig. 5.**

For variants 1 and 2 the unknown factor of stiffness increase is calculated using equation (3). For variants 3 and 4 the first unknown factor of stiffness increase is calculated using the equation (3), and the second factor is calculated based on minimum cost condition, using the equation (2). The history of GA-based search for a design with minimum cost of structure is shown in Fig. 4. A period of 500 generations is analyzed.

Obtained values of elastic modulus increase factors are shown in Fig. 5, where *1*—modulus increase factors for upper chord elements for each of $n = 12$ panels, *2*—modulus increase factors for lower chord elements, *3*—modulus increase factors for posts, *4*—modulus increase factors for braces.

Calculation results for 5 variants are given in Table 2.

Adopting the first variant as a baseline, the optimization results in 9.1% lower cost of structure. Genetic algorithm being heuristic, the GA-based result is not the most optimal, however, it is very close to the optimum. Besides, the application of GA-based optimization is limited to infusion-fabricated profiles only, as the infusion process allows for lay-ups consisting of various fabric types [11]. In case of pultrusion it is desirable to minimize a number of variants of varying modulus profiles of the same cross section [12].

The variant no. 4 of available accurate solutions is the closest to GA-optimized design, having 1/3 of upper and lower chord elements in the central section and 2/3 of posts and braces in end sections fabricated of profiles made with addition of carbon fibers. It is also worth noting that the increase of carbon fiber content in chord elements only results in degraded performance.

**Table 2**

| | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| Modulus increase coefficients, $\mu_i$ | $\tilde{\mu} = 1.193$ | $\tilde{\mu} = 1.430$ | $\tilde{\mu}_1 = 1.060$<br>$\tilde{\mu}_2 = 1.396$ | $\tilde{\mu}_1 = 1.171$<br>$\tilde{\mu}_2 = 1.368$ | $\mu_i$<br>see Fig. 5 |
| CFRP volume content, $k_i$ | $\tilde{k} = 0.62$ | $\tilde{k} = 0.362$ | $\tilde{k}_1 = 0.051$<br>$\tilde{k}_2 = 0.334$ | $\tilde{k}_1 = 0.144$<br>$\tilde{k}_2 = 0.310$ | $k_i$<br>see Eq. (5) |
| Weight of GFRP, kg | 4019 | 3637 | 4107 | 4317 | 4341 |
| Weight of CFRP, kg | 594 | 885 | 527 | 368 | 349 |
| Total weight, kg | 4613 | 4522 | 4634 | 4685 | 4690 |
| Cost, USD | 16125<br>(100%) | 1786<br>(+10.8%) | 15725<br>(–2.5%) | 14777<br>(–8.4%) | 14661<br>(–9.1%) |

## CONCLUSIONS

Computation methods to optimize the distribution of CFRP content within composite elements of truss bridge structures have been proposed. In order to solve the problem a modification of genetic algorithm [5], [6] has been developed combined with the methods of mathematical induction. A criterion of minimization of structural materials cost while satisfying stiffness requirements is used as an optimality criterion.

Based on modeling results, an optimized design of pedestrian truss bridge structure of composite materials has been developed, with span length of 36 m and walkway width of 3 m. The application of CFRP in amount of 7.4% of total weight of the structure resulted in 9.1% lower cost of structure while satisfying stiffness requirements.

## REFERENCES

1. Ushakov, A.E., Klenin, Yu.G., Sorina, T.G., Khairetdinov, A.Kh., and Safonov, A.A., Bridges structures made of composites, *Kompozity Nanostrukt.,* 2009, no. 3, pp. 25–37.
2. Kryzhanovskii, V.K., Burlov, V.V., Panimatchenko, A.D., et al., *Tekhnicheskie svoistva polimernykh materialov. Uchebno-spravochnoe posobie* (Technical Properties of Polymeric Materials. Tutorial-Reference Book), St. Petersburg: Professiya, 2003.
3. Sorina, T.G., Safonov, A.A., and Khairetdinov, A.Kh., Peculiarities of using carbon glass-reinforced plastic in pultrusion composite profiles for bridge engineering, *J. Mach. Manuf. Reliab.,* 2010, vol. 39, no. 1, p. 47.
4. Shcheglov, B.A. and Safonov, A.A., *Teoreticheskie osnovy i prikladnye zadachi tekhnologii kompozitov* (Theoretical Foundations and Applied Problems on Composites Technology), Moscow: LENAND URSS, 2015.
5. Gladkov, L.A., Kureichik, V.V., and Kureichik, V.M., *Geneticheskie algoritmy* (Genetic Algorithms), Moscow: Fizmatlit, 2006.
6. Kirsanov, M.N., Genetic algorithm for optimizing bar systems, *Stroit. Mekh. Raschet Sooruzh.,* 2010, no. 2, pp. 60–63.
7. Kirsanov, M.N., The way to analyze truss deflection of rectangular special coating, *Inzh.-Stroit. Zh.,* 2015, no. 1(53), pp. 32–38.
8. www.apatech.ru/pira.html.
9. Tin'kov, D.V., Optimal geometry of flat beam truss frame by considering material's linear creping, *Inzh.-Stroit. Zh.,* 2016, no. 1(61), pp. 25–32.
10. *SP* (Constructing Rules) *no. 35.13330.2011* (*SNiP* (Constructing Rules and Regulation) *no. 2.05.03-84): Bridges and Pipes*.
11. Safonov, A.A., Mathematical modeling for impregnation of reinforcing filler of fiberglasses during vacuum infusion, *J. Mach. Manuf. Reliab.,* 2010, vol. 39, no. 6, p. 568.
12. Safonov, A.A., Mathematical description of polymerization process under pultrusion drawing, *Probl. Mashinostr. Avtomatiz.,* 2005, no. 2, pp. 103–106.