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Calculating model of a frame type planar truss having an arbitrary number of panels

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ABSTRACT
Introduction. The subject of the study is the kinematic variability and deformations of a planar statically-determinate elastic truss with a horizontal bolt, lateral supporting trusses and a cross-shaped grid under the action of various types of static loads. The structure has three movable supports and one fixed support.
Objectives — derivation of formulas giving the dependence of the deflection of the structure in the middle of the span and the displacement of one of the three movable supports from the dimensions, load and number of panels; analysis of the kinematic variability and derivation of the analytical dependence of the forces in the rods of the middle of the span from the number of panels.
Materials and methods. Forces in the rods of the truss are calculated in symbolic form by cutting out nodes using the Maple symbolic and numeric computational environment. In order to calculate the deflection, the Maxwell – Mohr formula was used. Calculation formulas for the deflection and displacement of the support were derived using the induction method based on the results of analytical calculations of a number of trusses with a different number of panels in the crossbar and lateral support trusses. The special operators of the genfunc package for managing the rational generating functions of the Maple system were used to identify and solve the recurrence equations satisfied by the sequences of coefficients of the formulas for deflection and forces. It is assumed that all the rods of the truss have the same rigidity.
Results. Several variants of loads on the truss are considered. A combination of panel numbers is found in which the truss becomes kinematically variable. The phenomenon is confirmed by the corresponding scheme of possible velocities. All required dependences have a polynomial form by the number of panels. The curves of the dependence of the deflection on the number of panels and on the height of the truss are constructed in order to illustrate the analytical solutions.
Conclusions. The proposed scheme of a statically determinate truss is regular, allowing a fairly simple analytic solution of the deflection problem. The curves of the identified dependencies have significant areas of abrupt changes, which can be used in problems of optimising the design by weight and rigidity.

KEYWORDS: planar truss, frame, building frame, deflection, induction, Maple, analytical solution

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Расчетная модель плоской фермы рамного типа с произвольным числом панелей

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АННОТАЦИЯ
Введение. Представлен подход к кинематической изменяемости и деформации плоской статически определимой упругой фермы с горизонтальным ригелем и боковыми опорными фермами и крестообразной решеткой под действием различных видов статических нагрузок. Конструкция имеет три подвижные и одну неподвижную опору.
Цели — вывод формул зависимостей прогиба конструкции в середине пролета и смещения одной из трех подвижных опор от размеров, нагрузки и чисел панелей, анализ кинематической изменяемости, получение аналитических зависимостей усилий в стержнях середины пролета от числа панелей.
Материалы и методы. Наиболее распространенным подходом определения усилия в стержнях фермы является вычисление в символьной форме методом вырезания узлов с использованием системы компьютерной математики Maple. Для вычисления прогиба используется формула Максвелла – Мора. Методом индукции по результатам аналитических расчетов ряда ферм с различным числом панелей в ригеле и боковых опорных фермах выводятся расчетные формулы для прогиба и смещения опоры. Специальные операторы пакета genfunc для управления рациональными производящими функциями системы Maple дают возможность найти и решить рекуррентные уравнения, которым удовлетворяют последовательности коэффициентов формул для прогиба и усилий. Принимается одинаковая жесткость всех стержней фермы.
Результаты. Одним из преимуществ рассмотренных вариантов нагрузок на ферму является обнаружение сочетания чисел панелей, при котором ферма становится кинематически изменяемой. Явление подтверждено соответствую-

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Vestnik MGSU • Proceedings of Moscow State University of Civil Engineering • Volume 13 Issue 10, 2018

щей схемой возможных скоростей. Все искомые зависимости имеют полиномиальную форму по числам панелей. В качестве иллюстрации аналитических решений построены кривые зависимости прогиба от числа панелей и от высоты фермы.

Выводы. Предложенная схема статически определимой фермы является регулярной и допускает достаточно простое аналитическое решение задачи о прогибе. Кривые найденных зависимостей имеют участки значительных скачкообразных изменений, что может быть использовано в задачах оптимизации конструкции по весу и жесткости.

КЛЮЧЕВЫЕ СЛОВА: плоская ферма, рама, каркас здания, прогиб, индукция, Maple, аналитическое решение

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INTRODUCTION

Planar trusses are often used as the bearing components of spatial structures. In many cases the primary goal of the designers of industrial buildings, workshops and hangars is to increase the rigidity of the frames [1–3]. Thus, the development of new structural schemes for trusses that ensure the optimal rigidity and stability of construction elements is an urgent task. Simple and reliable tests used in analytic calculations of building structures may on the one hand represent an alternative to numerical studies, but, on the other hand, in many ways can be seen to complement numerical approaches.

We propose a scheme of a statically-determinate, articulated frame with a cross-shaped grid (Fig. 1). The truss is one of the regular types, based around a periodicity cell. The length of the crossbar is $2n$ panels; the height of the truss — m. The height of the truss is $2(m+1)h$. The truss contains $n_s = 8(n+m)+26$ rods, including five rods comprising simulated supports. The aim is to obtain an analytical expression for the deflection, depending on the number of panels in the crossbar and racks. The derivation of this dependence assumes the use of an induction method across two parameters, which considerably complicates the solution. However, formulas with two independent parameters regulating the scheme of the truss have a much wider scope: from low frames with a large span to high tower-type trusses. Previously, the majority of comparable solutions for planar statically-determinate regular trusses were obtained by induction using one parameter only [4–8]. By applying induction across two parameters, a solution is obtained for spatial coverage [9] both for a planar frame [10] and for a cable-stayed truss [11].

CALCULATION

In order to calculate the deflection in the analytical form, it is first necessary to know the expressions for the forces in the lattice of the truss. The solution is obtained using the Maple symbolic mathematics computer program [12]. This program includes a method for cutting out nodes. To implement the method, the coordinates of the hinges are entered in the program. The connection structure of nodes (hinges) and rods is specified by special vectors representing the numbers of the ends of the rods. The matrix of the equilibrium equations for nodes is formed from the direction cosines of forces, determined from the given geometry of the structure and the order in which the rods are joined [9–11]. The odd rows of the numbers in the matrix correspond to the projections of the forces on the x-axis, while the even rows correspond to the projections onto the y-axis. The right-hand side of the system of equilibrium equations comprises a vector in which components of the load on the nodes are recorded. Solving the system of equations

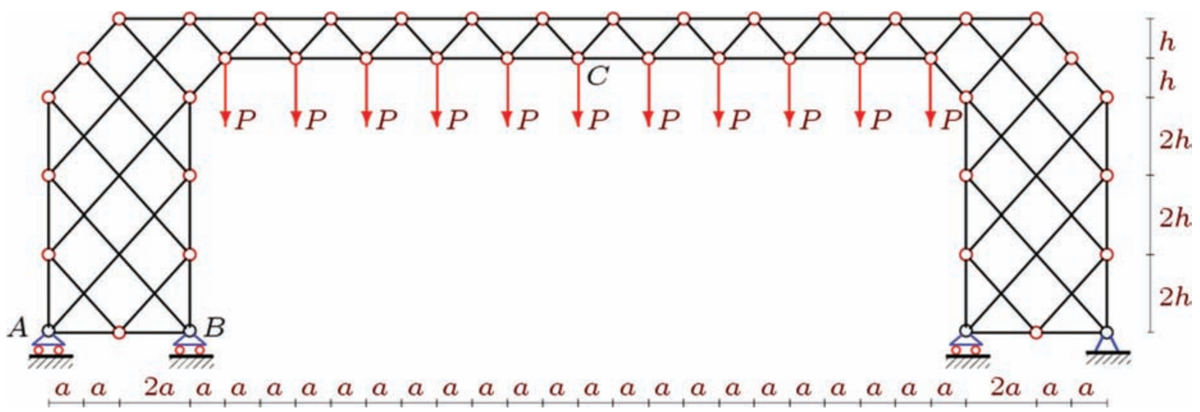

Fig. 1. Diagram of the truss, load on the lower belt, m = 3, n = 5

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in symbolic form with the given numbers of panels m and n gives the forces applying in all rods, including support rods.

The first calculations of forces for different numbers of panels showed that, for an odd number of panels in the crossbar, the determinant of the system of equations of node equilibrium becomes zero, which corresponds to the kinematic variability of the truss. This is confirmed by the derived scheme for the distribution of virtual node velocities (Fig. 2). Vectors of velocities do not contradict the laws of kinematics; in particular, the theorem on the projections of velocities is fulfilled for all rods, i.e. the projections of the velocities of the rod ends on the axis of the rod (with the sign of the projection) are equal. The relationship between the velocities can be determined by considering the instantaneous centres of the velocities of the rods: $u/h = v/a$.

The deflection is calculated using the Maxwell-Mohr formula

$$\Delta = P\sum_{j=1}^{n_s-5}\frac{S_j s_j l_j}{EF}, \qquad (1)$$

where E is the modulus of elasticity of the rods, F is the cross-sectional area, l_j and S_j are the length and force respectively in the j-th rod from the action of the load, s_j is the force from a single vertical force applied to the central node C in the lower belt of the crossbar. The calculation is performed for all the pins in the truss, except for the five support pins, which are assumed to be rigid. It turns out that the form of the solution does not change for trusses with different numbers n and m:

$$\Delta = P\left(C_1a^3 + C_2c^3 + C_3h^3\right)\Big/\left(2h^2EF\right), \qquad (2)$$

where $c = \sqrt{a^2+h^2}$. Solutions differ only in coefficients before cubes a^3, c^3 and h^3. Using the **rgf_findrecur** operator of the Maple package **genfunc** by induction on n, for $m = 1, 2, 3$ ... from the solution of the recurrence equation

$$C_{1,n} = 5C_{1,n-1} - 10C_{1,n-2} + 10C_{1,n-3} - 5C_{1,n-4} + C_{1,n-5}$$

a sequence of coefficients is obtained

$$C_1 = (20n^4 + 40n^3 + 34n^2 + 38n + 15)/3,$$
$$C_1 = (20n^4 - 40n^3 + 10n^2 + 40n + 12)/3,$$
$$C_1 = (20n^4 + 40n^3 + 34n^2 + 38n + 15)/3,$$
$$\ldots$$

In order to solve the recurrence equation, the **rsolve** operator is used. The periodic regularity of the variation as a function of m coefficients for powers of n is fairly obvious. As a result, we have the general case:

$$C_1 = \Big(20n^4 - 40n^3(-1)^m + \left(22 - 12(-1)^m\right)n^2 +$$
$$+\left(39 + (-1)^m\right)n + 3\left(9 - (-1)^m\right)\big/2\Big)\Big/3.$$

Similarly, we derive other expressions:

$$C_2 = 2n^2 + 2n\left(\left(2-(-1)^m\right)m + 3 - (-1)^m\right) +$$
$$+\left(2-(-1)^m\right)m - (-1)^m + 3,$$
$$C_3 = 5m(2n+1) +$$
$$+(1+2n)\left(2\cos\varphi - 2\sin\varphi - (2-m)\cos 2\varphi\right),$$
$$\varphi = \pi m/2.$$

At the same time, expressions were obtained for the forces in the middle rods of the span

$$O = -Pa\left(2n^2 - (2n+1)(-1)^m\right)\big/(2h),$$
$$D = Pc/(2h),$$
$$U = Pa\left(n^2 - n(-1)^m - \left(1+(-1)^m\right)\big/2\right)\big/h.$$

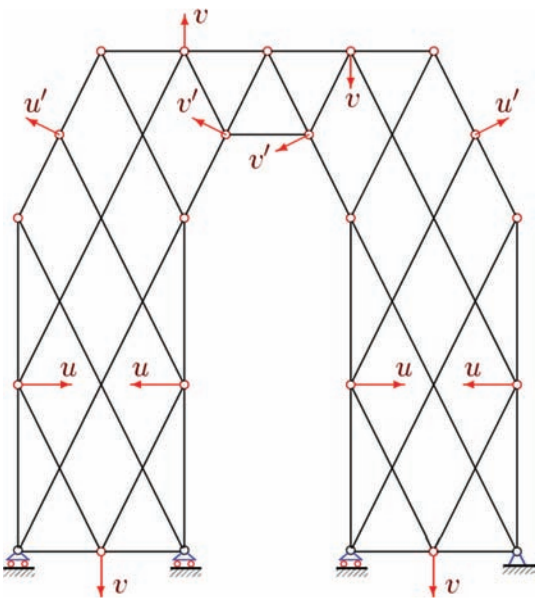

Fig. 2. Scheme of possible velocities of the variable truss, with one panel in the crossbar and $m = 2$

Вестник МГСУ ISSN 1997-0935 (Print) ISSN 2304-6600 (Online) Том 13 Выпуск 10, 2018
Vestnik MGSU • Proceedings of Moscow State University of Civil Engineering • Volume 13 Issue 10, 2018

In contrast to beam trusses, it is also important to know the dependence of the horizontal displacement of the mobile support on the dimensions, load and number of panels in frame structures. Using the same formula (1), in which this time s_j is the forces from the unit horizontal force applied to the support A, we obtain the following expression

$$\delta_A = 2P\left(A_1a^3 + A_2c^3 + A_3h^3\right)\big/\left(haEF\right), \qquad (3)$$

where

$$A_1 = \left(-8n^3 + 12n^2(-1)^m + \right.$$
$$+\left(9\cos 2\varphi + 17 - 6\cos\varphi + 6\sin\varphi\right)n +$$
$$\left.+3\left(2\sin\varphi - 2\cos\varphi + 5 + \cos 2\varphi\right)/2\right)\big/6,$$

$$A_2 = \left((4 - 2m\cos 2 + 2m)n + 2 - m\cos 2 + m\right)\big/4,$$

$$A_3 = \left((2n+1)\left((-1)^m + 3\right)m^2 + \right.$$
$$\left.+8(2n+1)m + 3(2n+1)\left(1-(-1)^m\right)\right)\big/8.$$

Reaction of supports are:

$$Y_A = P(2n+1)\left(1+(-1)^m\right)\big/8, \; Y_B = P(n+1/2) + Y_A.$$

The algorithm used to derive the formulas makes it easy to switch the load to the truss. Consider the effect of the load on the upper frame belt (Fig. 3).

The coefficients in (2) take the form:

$$C_1 = \left(20n^4 - 40n^3(-1)^m - 20n^2 + \right.$$
$$\left.+\left(43(-1)^m + 27\right)n + 12\left(1+(-1)^m\right)\right)\big/3,$$
$$C_2 = 2n^2 + 2\left((2-(-1)^m\right)m -$$
$$-(-1)^m + 3)n + 2\left(1+(-1)^m\right),$$
$$C_3 = 2(m\cos 2\varphi - 2\cos 2\varphi + 2\cos\varphi - 2\sin\varphi + 5m)(n+1).$$

Forces applying in rods can be calculated from formulas

$$O = -Pa\left(n - 2(-1)^m\right)\left(n+(-1)^m\right)\big/h,$$
$$D = 0,$$
$$U = Pa\left(n+(-1)^m\right)\left(n-2(-1)^m\right)\big/h.$$

Reaction of supports are:

$$Y_A = -Pn\left(1+(-1)^m\right)\big/4, \; Y_B = P(n+2) - Y_A.$$

The coefficients in (3) for calculating the displacement of the support under this loading are:

$$A_1 = \left(-4n^3 + 6n^2(-1)^m + \right.$$
$$\left.n(41 + 3\cos 2 - 6\cos + 6\sin)/2 + 12\right)\big/3,$$
$$A_2 = n\left(2+\left(1-(-1)^m\right)m\right)\big/2 + 1,$$
$$A_3 = \left(\left(n\left((-1)^m + 3\right) + 4\right)m^2 + \right.$$
$$\left.+8(n+1)m + (3n+2)\left(1-(-1)^m\right)\right)\big/4.$$

In conclusion, we also give the formulas obtained for the case of loading the frame with a concentrated force (Fig. 4). The solution is simpler, with the recurrence equations having a smaller degree. The time of performing symbolic transformations is much shorter.

The coefficients in (2) have the form

$$C_1 = \left(16n^3 - 24(-1)^m n^2 + 14n + 15 - 3\left(1+(-1)^m\right)/2\right)\big/3,$$
$$C_2 = 2n + \left(2-(-1)^m\right)m - (-1)^m + 3,$$
$$C_3 = 5m + m\cos 2\varphi -$$
$$-2\cos 2\varphi + 2\cos\varphi - 2\sin\varphi.$$

The forces in the rods and the reactions of the supports are obtained in the following form:

$$O = -Pa\left(2n-(-1)^m\right)\big/(2h),$$
$$D = Pc/(2h),$$
$$U = Pa\left(n - \left(1+(-1)^m\right)/2\right)\big/h,$$
$$Y_A = -P\left(1+(-1)^m\right)\big/8,$$
$$Y_B = P\left(1+\left(1+(-1)^m\right)/4\right)\big/2.$$

The verification of the obtained solutions was carried out in the numerical mode of the same program for arbitrary combinations of the numbers m and n. A linear combination of the three solutions can be used to calculate rather complicated cases of loading. With the load evenly distributed over the nodes, it is permissible to simulate a constant load from the weight of the frame

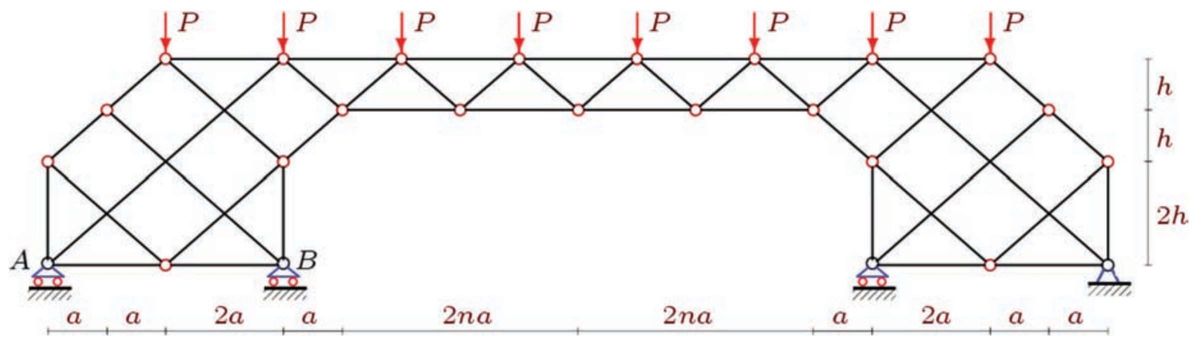

Fig. 3. Load on the upper frame belt, $n = 2$, $m = 1$

Вестник МГСУ ISSN 1997-0935 (Print) ISSN 2304-6600 (Online) Том 13 Выпуск 10, 2018
Vestnik MGSU • Proceedings of Moscow State University of Civil Engineering • Volume 13 Issue 10, 2018

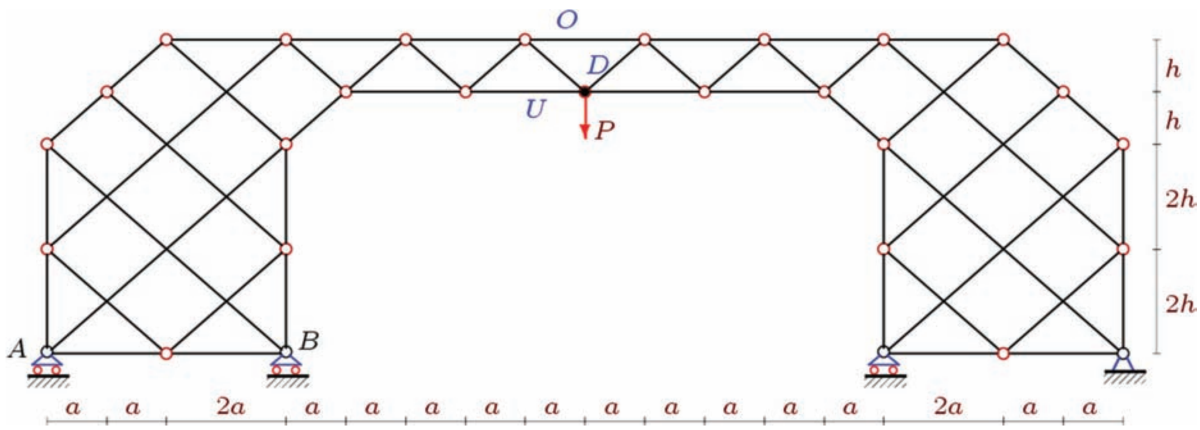

Fig. 4. Frame under the action of concentrated force, $n = 2$, $m = 2$

Fig. 5. 3D model of the truss as part of the building of the industrial building, $n = 5$, $m = 4$. Horizontal links are not shown

itself; with a concentrated load, the load from the crane in an industrial building is applied. The frame can be included in the spatial construction of the building (frame of a warehouse or factory workshop). Individual frames are connected by horizontal links (Fig. 5).

ANALYSIS

The curves of the dependence of the dimensionless deflection on the number of panels (Fig. 6) in the condition of constancy of the span of the structure $L = 60$ m, $a = L/(4n)$ show that, starting from a particular value of n, the relative deflection increases almost linearly. Here is denoted: $\Delta' = \Delta EF/(P_{sum}L)$, $P_{sum} = P(2n+4)$. The point of the break at the beginning of the graph is accounted for by unrealistically long panels $a = L/(4n) = 60/8 = 7.5$ m and has no practical meaning to optimise rigidity. However, the very presence of such a point suggests that, with certain combinations of sizes and areas of the rod section, it is possible to find a condition under which the rigidity of the structure is optimal.

The slope of the asymptote of the curves on the graph gives the following limit $\lim\limits_{n\to\infty} \Delta'/n = h/(2L)$.

The dependence of the deflection on the number of panels m in height is more complicated (Fig. 7). Curves constructed at a fixed frame height show strong kinks. This feature of the solution can be used to optimise the rigidity of the structure. Without changing the height of the truss, but only changing the number of panels in height, it is possible to increase the rigidity by a factor of *two*. This can be seen in Fig. 7 from the curve constructed at $H = 30$ m at the points $m = 3$ and $m = 4$.

The curves of the dependence of the deflection on the height of the truss (Fig. 8) show that this function has a pronounced minimum. It follows from the self-intersection of the curves that the order of the curves with an increase in the number of panels m is different for different heights. For small heights ($h < 7$ m), the hard-

Вестник МГСУ ISSN 1997-0935 (Print) ISSN 2304-6600 (Online) Том 13 Выпуск 10, 2018
Vestnik MGSU • Proceedings of Moscow State University of Civil Engineering • Volume 13 Issue 10, 2018

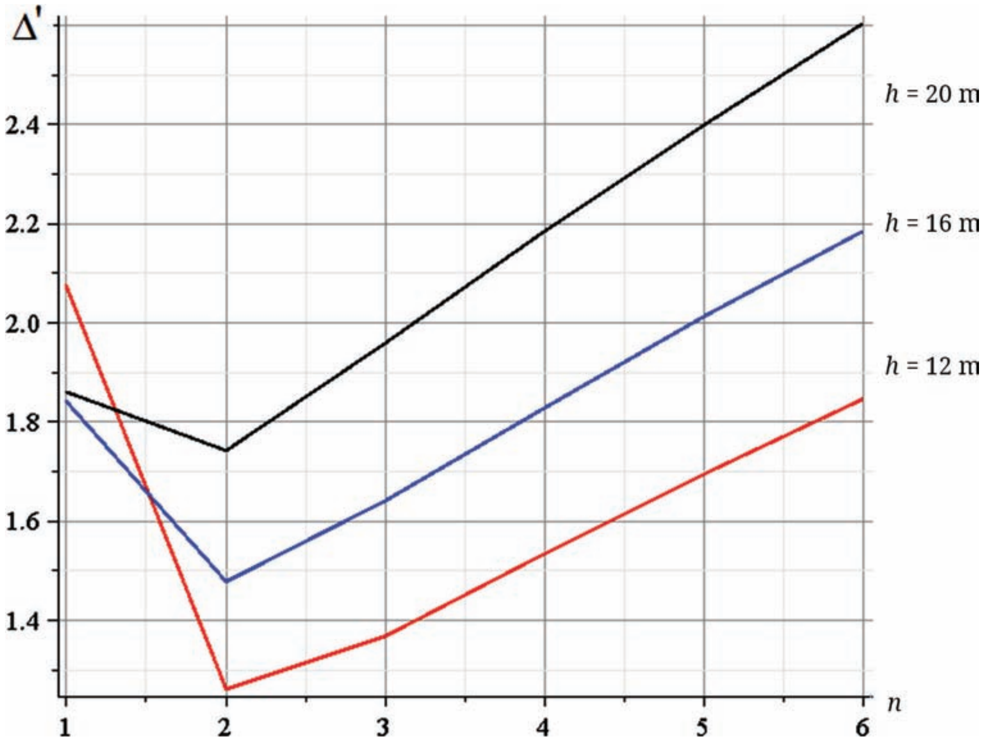

Fig. 6. Dependence of the deflection on the number of panels, $m = 2$

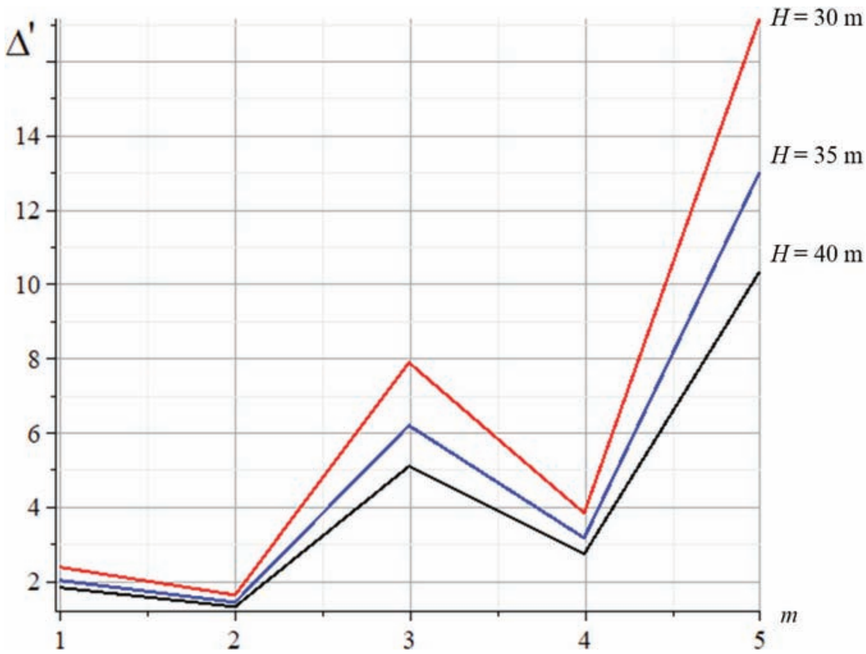

Fig. 7. Dependence of the deflection on the number of panels in height, $n = 3$

Вестник МГСУ ISSN 1997-0935 (Print) ISSN 2304-6600 (Online) **Том 13 Выпуск 10, 2018**
Vestnik MGSU • Proceedings of Moscow State University of Civil Engineering • **Volume 13 Issue 10, 2018**

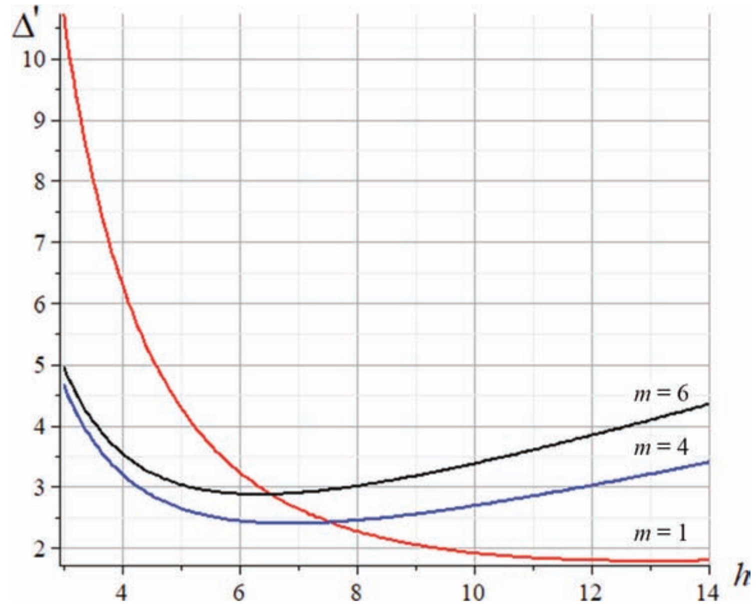

Fig. 8. Dependence of the deflection on the height of the truss, $n = 4$, $L = 60$ m

Вестник МГСУ ISSN 1997-0935 (Print) ISSN 2304-6600 (Online) Том 13 Выпуск 10, 2018
Vestnik MGSU • Proceedings of Moscow State University of Civil Engineering • Volume 13 Issue 10, 2018

est is the truss at $m = 4$; for greater heights, the smallest deflection is in the truss with one panel along the height of the structure. The extreme point (the minimum on the curve) moves to the right with decreasing number m, reaching unrealistically large values $h \approx 7$ m for $m = 1$.

CONCLUSIONS

The search for schemes for statically-determinate regular constructions, begun by R.G. Hutchinson, N.A. Fleck [13, 14], is continuing. In this paper, another frame type truss is proposed, inductive analysis carried out and the derivation of compact formulas for calculation of deflection and forces, depending on the number of panels, presented. In addition to solving static problems and the observed effects of an abrupt change in rigidity, a potentially dangerous hidden construction feature of this structure was found for this design. With an odd number of panels in the crossbar, regardless of the size of the truss, the number of panels in height and the type of load, the determinant of the system of equilibrium equations degenerates, indicating the kinematic variability of the truss. In the general case, this feature can be overlooked in the numerical analysis beyond the calculation error. In addition, in a real execution of a truss of this kind have rigid or semi-rigid connections instead of hinges and the kinematic variation of the hinge model is not apparent. However, this does not reduce the potential danger of the found effect; consequently, a reasonable designer will avoid using a scheme with an odd number of nodes in a crossbar of this design.

The resulting formulas for deflection can be used in practice to estimate the deflection of the projected trusses; the method itself is quite applicable to other planar and spatial regular constructions. A survey of analytical solutions for planar trusses is given in [4–6]. Questions of the theory of regular building structures were considered in [15–20]. Inductive analysis and derivation of formulas for the deflection of some planar trusses carried out in [21–25].

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