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DEFORMATIONS AND NATURAL FREQUENCY SPECTRUM OF A PLANAR REGULAR TRUSS WITH A TRIANGULAR LATTICE

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The formula for the dependence of the deflection of a statically determined frame-type truss on the number of panels is derived by induction in the Maple computer mathematics system. A uniform and concentrated load on the upper belt is considered. A picture of the distribution of forces on the truss rods is given. The Dunkerley method is used to find a lower analytical estimate of the first oscillation frequency under the assumption that the truss mass is uniformly distributed over the nodes. Generalization of the solution to an arbitrary number of panels is carried out by induction. The solution is compared with the numerical solution for the entire frequency spectrum found using the Maple eigenvalues operator. The high accuracy of the obtained estimate is noted, which grows with the increase in the number of panels. In the set of spectra of regular trusses of various orders, spectral isolines and spectral constants are found. The error of the analytical estimate, which is a formula with coefficients in the form of polynomials in the number of panels not higher than the sixth order, does not exceed 24%. When the number of panels is more than eight, the estimation error is less than 5%.

Key words: truss, frame, Dunkerley method, oscillations, fundamental frequency, induction, isoline, Maple

Introduction

In most static calculations of building structures, numerical methods are used [1-3]. The development of methods of symbolic mathematics makes it possible to obtain analytical results as well. Such solutions are often reduced to some algorithms implemented in the systems Maple, Mathematica, Derive, Reduce, etc., but do not give compact calculation formulas [4,5]. The most difficult task is the derivation of simple calculation formulas that are valid not for anyone's construction, but a certain class. These classes include regular constructions containing periodically repeating elements in their structure. In trusses, for example, such a group can be a separate panel. The problems of the existence of statically determinate regular trusses were dealt with by Hutchinson R.G. and Fleck N.A. [6,7]. Some solutions for deflections of planar trusses of a regular type were obtained in [8-12], for spatial ones in [13]. These solutions are obtained by the inductive generalization of a series of analytical solutions with obtaining common terms of sequences of coefficients in the calculation formulas. A set of solutions for truss deflections and shifts of supports under various loads of beam, frame, arch, and cantilever trusses is contained in the reference book [14].

As a rule, in the dynamic calculations of building structures, the spectra of natural frequencies are used, and above all, the first frequency. Much less is known about analytical solutions for truss oscillation frequencies. This is primarily because the frequency equations in such problems are algebraic equations that have a high order and are not solvable in elementary functions. However, there are approximate estimates for the first frequency, which allow not only analytical solutions but also their generalizations to an arbitrary order of the regular construction [15–19]. An exact solution for the lower bound for the first frequency of a planar truss was obtained in [20].

This paper proposes analytical solutions for planar truss deflection and first frequency estimation. The frequency spectrum of natural oscillations is investigated

Calculation of forces and deflections

A frame-type truss (Fig. 1) has $2n$ panels in the crossbar and two panels each in the side parts. Truss panels are made up of equilateral triangles with side a. Truss height is $3h = 3a\sqrt{3}/2$. The left support of the truss is a movable hinge, the right one is fixed. The total number of bars, including the three bars modeling the supports, is $\eta = 8n+22$.

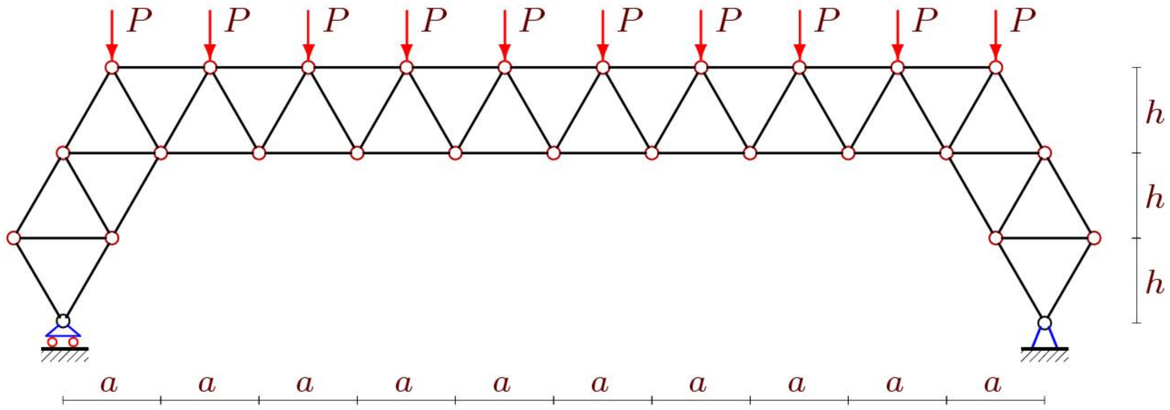

Fig. 1. The truss scheme n=4

Let us calculate the forces in the truss rods from the action of a uniform nodal load on the crossbar of the upper chord. All calculations and transformations will be carried out in analytical form in the Maple system according to the program [21]. We number the nodes (Fig. 2) and set their coordinates. The origin of coordinates is located in the movable support A. Here is the corresponding fragment of the program in the Maple language:

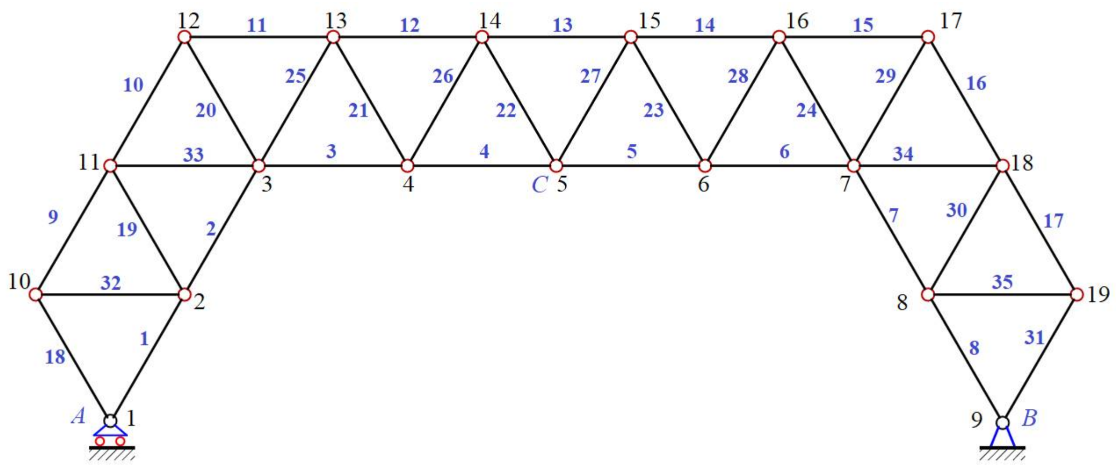

Fig. 2. Numbering of nodes and rods of the truss, n=2

```
for i to 3 do
  x[i]:=a*i/2-a/2;y[i]:=h*i-h;
  x[i+2*n+2]:=a*i/2+a/2+2*n*a;   y[i+2*n+2]:=3*h-h*i;
end:
for i to 2*n-1 do
  x[i+3]:=a*i+a;   y[i+3]:=2*h;
end:
```

```
 for i to 2 do
  x0:=(x[i+1]+x[i])/2;            y0:=(y[i+1]+y[i])/2;
  x[i+2*n+5]:=x0-h* sqrt(3)/2;    y[i+2*n+5]:=y0+h/2;
  x0:=(x[i+3+2*n]+x[i+2+2*n])/2; y0:=(y[i+3+2*n]+y[i+2+2*n])/2;
  x[i+9+4*n]:=x0+h*sqrt(3)/2;     y[i+9+4*n]:=y0+h/2;
 end:
 for i to 2*n+2 do
  x[i+2*n+7]:=-a/2+a*i; y[i++2*n+7]:=3*h;
end:
```

The order of connection of rods at the nodes will be specified by special lists **T[i]**, where i is the number of the rod, containing the numbers of the ends of the corresponding rods. For example, the lower belt is set like this:

```
for i to 2*n+4 do T[i]:=[i,i+1]; end:
```

The supports are modeled by rods of length f. Based on these data, the coefficients of the matrix of equilibrium equations of nodes in projections on the coordinate axes are calculated. Vector **B** on the right side of the system in odd elements contains projections of nodal loads on the *x*-axis, in even elements — on the vertical *y*-axis. In this case:

```
for i from 2*n+8 to 4*n+9 do B[2*i]:=-P; end;
```

Let us present the results of the force calculation. The distribution of forces on the rods $a = 3\,\text{m}, f = 1\,\text{m}$ is shown in Figure 3. Compressed rods are highlighted in blue, stretched rods are highlighted in red. The thickness of the segments of the rods is conditionally proportional to the modules of the corresponding forces. The force value is related to the value of the nodal load *P* and rounded up to two digits.

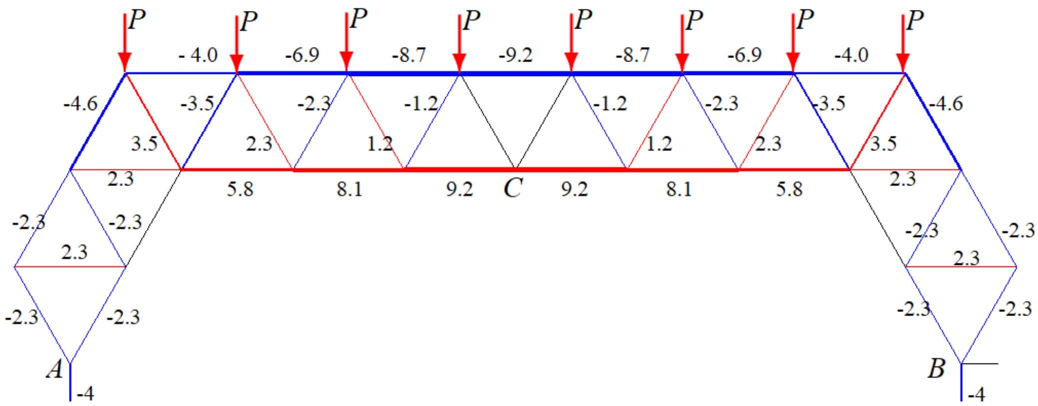

Fig. 3. Distribution of forces in the rods, *n*=3

The lower crossbar, as expected, is stretched, the upper one is compressed. The upper rods of the side parts in the lower chord are unloaded.

Deflections

Truss deflection is calculated using the Maxwell – Mohr formula

$$\Delta_n = \sum_{i=1}^{\eta} S_i^{(1)} S_i^{(P)} l_i / (EF).$$

Here $S_i^{(1)}$— is the force in the rod i from the action of a single vertical force on the node in which the deflection is measured, l_i— is the length of the rod i, E is the modulus of elasticity of the rods, F is their cross-sectional area. The rigidity of the rods is assumed to be the same.

Solving the deflection problem in analytical form for n=1, 2, 3, ..., we successively obtain:

$$\Delta_1 = P(18a + 2f)/EF,$$
$$\Delta_2 = P(63a + 3f)/EF,$$
$$\Delta_3 = P(172a + 4f)/EF,$$
$$\Delta_4 = P(1175a/3 + 5f)/EF,$$
$$\Delta_5 = P(782a + 6f)/EF,....$$

If the common term of the sequence of coefficients at f is obvious, then the common term of the sequence 18, 63, 172, 1175/3, 782 is found using the operators of the Maple system.

Thus, we have the following dependence of the deflection on the number of panels

$$\Delta_n = P((5n^4 + 20n^3 + 43n^2 + 61n + 33)a/9 + (n+1)f)/EF.$$

Similarly, we find the deflection of the truss from the action of the vertical concentrated force P at the node C

$$\Delta_n = P((8n^3 + 24n^2 + 49n + 48)a/9 + f)/(2EF).$$

Under the action of a vertical load, the left movable bearing receives a displacement δ. If in formula (1) we understand $S_i^{(1)}$ the forces in the rods from the action of a single horizontal force on node A, then as a result of induction on ten analytical solutions, we obtain the following value of the horizontal displacement of support A from the action of a uniform vertical load

$$\delta_n = 2Pa\sqrt{3}(10n^3 + 30n^2 + 59n + 27))/EF.$$

The spectrum of natural frequencies

The inertial properties of the truss are modeled by the masses concentrated in the nodes, the masses of the rods are neglected. Let us assume that the vibrations of the loads are vertical. The number of degrees of freedom of the system in this case is equal to the number of nodes $N = 4n + 11$.

The differential equations for the oscillations of a system of N weights have the form:

$$\mathbf{J}_N \ddot{\mathbf{Y}} + \mathbf{D}_N \mathbf{Y} = 0, \qquad (1)$$

where $\mathbf{D}_N$ is the stiffness matrix, $\mathbf{Y} = [y_1, y_2, ..., y_N]^T$ is the vector of vertical displacements of loads, $\mathbf{J}_N = m\mathbf{I}_N$ is the diagonal matrix of inertia, $\mathbf{I}_N$ is the identity matrix, $\ddot{\mathbf{Y}}$ is the vector of accelerations of nodes with masses. The inverse of the stiffness matrix $\mathbf{D}_N$ is the compliance matrix $\mathbf{B}_N$, whose elements are calculated using the Mohr integral:

$$b_{i,j} = \sum_{\alpha=1}^{\eta} S_\alpha^{(i)} S_\alpha^{(j)} l_\alpha / (EF). \qquad (2)$$

Here $S_\alpha^{(i)}$— is the force in the rod α from the action of a unit vertical force at node i, l_α— is the length of the rod α. Multiply (1) on the left by the matrix $\mathbf{B}_N$. Taking into account the identity $\ddot{\mathbf{Y}} = -\omega^2 \mathbf{Y}$, where is ω — the natural frequency of oscillations, we obtain the equation $-\omega^2 m \mathbf{B}_N \mathbf{Y} + \mathbf{Y} = 0$.

The problem was reduced to the problem of the eigenvalues $\lambda = 1/(m\omega^2)$ of the matrix $\mathbf{B}_N$. In the general case, such a problem can only be solved numerically. The approximate solution according to the Dunkerley method [22] for the lower estimate of the first vibration frequency ω_D is expressed in terms of the vibration frequencies of individual loads in the marked nodes:

$$\omega_D^{-2} = \sum_{k=1}^{N} \omega_k^{-2} \ , \qquad (3)$$

where ω_k is the partial frequency of oscillations of the mass m located at the truss node. When calculating the partial frequency, equation (1) takes the form of a scalar form: $m\ddot{y}_k + d_k y_k = 0$, where d_k is the stiffness coefficient, y_k is the mass displacement, and $\ddot{y}_k$ is the acceleration. Hence, the oscillation frequency of one load (partial frequency) has the form: $\omega_k = \sqrt{d_k / m}$. The stiffness coefficient is calculated using the Mohr integral: $\delta_k = 1/d_k = \sum_{\alpha=1}^{\eta} \left(\tilde{S}_\alpha^{(k)}\right)^2 l_\alpha / (EF)$. It is denoted here: $\tilde{S}_\alpha^{(k)}$ — forces in the rod with the number α from the action of a single vertical force applied to the node where the mass with the number α is located. From (3) follows:

$$\omega_D^{-2} = m \sum_{k=1}^{N} \delta_k = m\Delta_n. \qquad (4)$$

Sequentially calculating the coefficient Δ_n, we obtain

$$\begin{aligned}
\Delta_1 &= \frac{25(4a+f)}{2EF}, \\
\Delta_2 &= \frac{1638a+181f}{12EF}, \\
\Delta_3 &= \frac{10709a+567f}{32EF}, \\
\Delta_4 &= \frac{217240a+6111f}{300EF}, \\
\Delta_5 &= \frac{151993a+2487f}{108EF}, \ldots
\end{aligned} \qquad (5)$$

General view of the solution for the coefficient Δ_n:

$$\Delta_n = (C_1 a + C_2 f) / (EF(n+1)^2) \qquad (6)$$

We find the coefficients in this formula by generalizing the corresponding coefficients in (5). Direct application of the Maple operators does not work in this case. In order to find the common members of the sequences of coefficients for *a* and *f*, it was necessary to choose the form of their denominators depending on *n*. As a result, we have

$$\begin{aligned}
C_1 &= (512n^6 + 3072n^5 + 9680n^4 + 21600n^3 + 34133n^2 + 29103n + 9900)/5490, \\
C_2 &= (32n^3 + 180n^2 + 265n + 123)/12.
\end{aligned} \qquad (7)$$

From here, taking into account (3) and (4), we obtain the final formula for the lower limit of the first natural oscillation frequency of the truss:

$$\omega_D = (n+1)\sqrt{\frac{EF}{m(C_1 a + C_2 f)}}. \quad (8)$$

To estimate the approximate analytical solution (8) with coefficients (7), we compare (8) with the first frequency of the entire frequency spectrum obtained from the numerical solution of the problem of oscillation of a system with N degrees of freedom. To find the eigenvalues of a matrix $\mathbf{B}_N$, you can use the Maple *Eigenvalues* operator. The graph (Fig. 4) compares the dependence curves of the first frequency ω_1 obtained numerically and ω_D by formula (8). The curves are very close, and after $n = 5$ they practically coincide. It is assumed: $EF = 0.5 \cdot 10^6 \text{kH}$, m = 400 kg, a = 2 m, f = 1 m. The relative error $\varepsilon = (\omega_1 - \omega_D)/\omega_1$ decreases with an increase in the number of panels (Fig. 5). For more rigid supports (supports modeled by rods of shorter length f), the error is slightly larger.

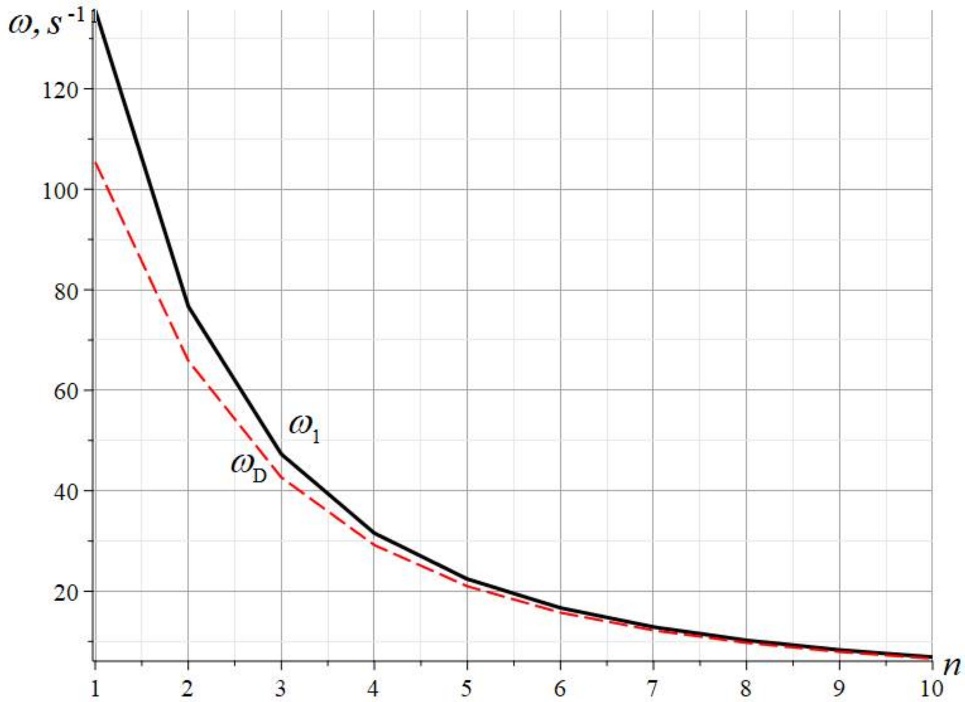

Fig. 4. Frequency dependence on the number of panels

The resulting formulas can be used to estimate the fundamental frequency of truss oscillations with a very large number of rods. The accuracy of the numerical calculation with an increase in the number of panels naturally (due to the accumulation of rounding errors) decreases, while the accuracy of the obtained analytical solution increases.

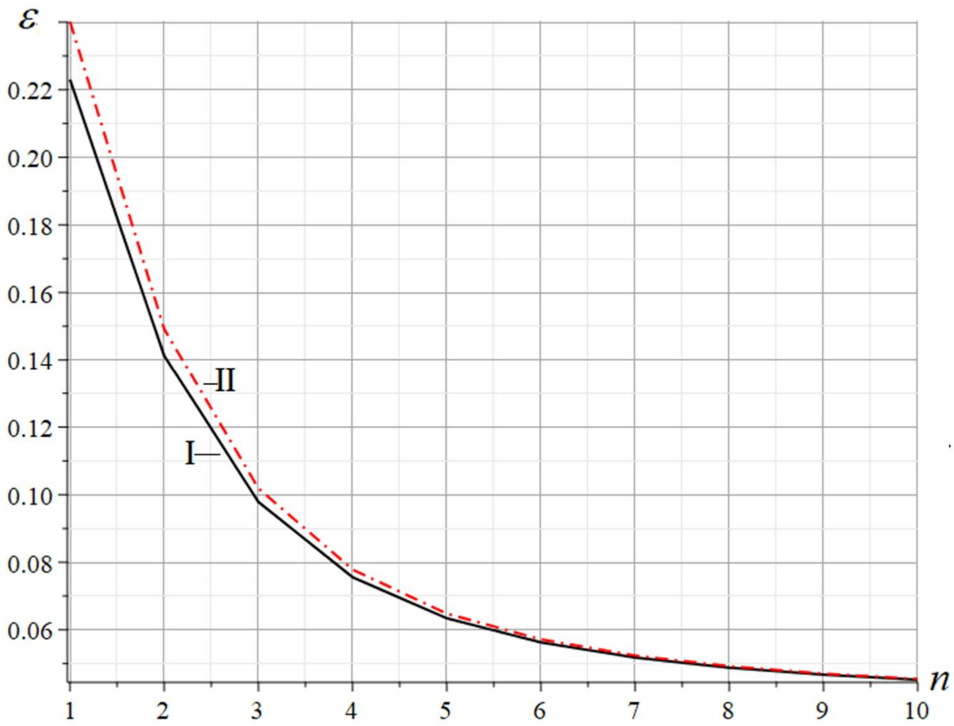

Fig. 5. Dunkerley estimation error depending on the number of panels.
I — f = 1m; II — f = 0.25 m

The solution error, depending on the number of panels, varies from 4% to 24% for a truss with one panel in half a crossbar or 14% with two panels.

The upper estimate of the lowest frequency by the Rayleigh method is also known [18, 19]. The Rayleigh method gives even greater accuracy, but the analytical solution is too cumbersome.

Isolines of natural frequency spectra

To assess the accuracy of the analytical solution according to Dunkerley, all frequencies of the spectra of trusses of various orders were calculated. During the analysis of the spectra, a feature of the frequency distribution over the spectra was noticed (Fig. 6). The spectra of trusses with the number of panels n from 1 to 10 in half of the crossbar are constructed for the same truss parameters as the graphs in Figure 4. The points of the spectrum (frequency) of each truss are conditionally connected by broken lines. There is some order in the frequency distribution pattern. First, the higher frequencies of each spectrum are almost the same and form four distinct groups. The first group of two higher frequencies is located around the frequency $1420\,s^{-1}$, the second — around the frequency $1190\,s^{-1}$. Second, each truss (except n = 1) has the same number of multiple frequencies. *Spectral isolines* can be distinguished in the spectra (Fig. 7). The isoline with the number {1} consists of the highest (last in the spectrum, ordered in ascending order) truss frequencies of various orders. The isolines with the number {2} connect the penultimate frequencies in the spectrum. In the general case, the isoline {v} of the set of regular trusses of orders consists of frequencies $\omega_{N+1-\nu}$, where $N = 4n + 11$ is the number of degrees of freedom of the truss of order n or the number of the last frequency in the spectrum.

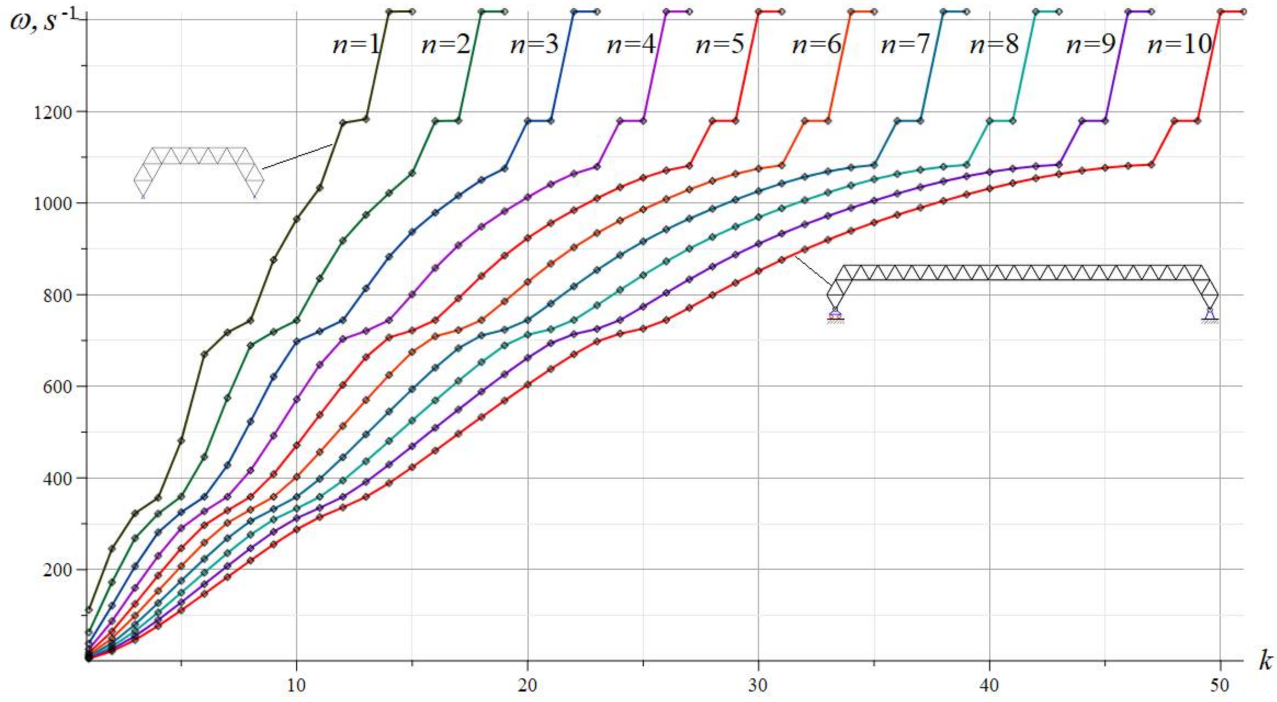

Fig. 6. Frequency spectra of regular trusses

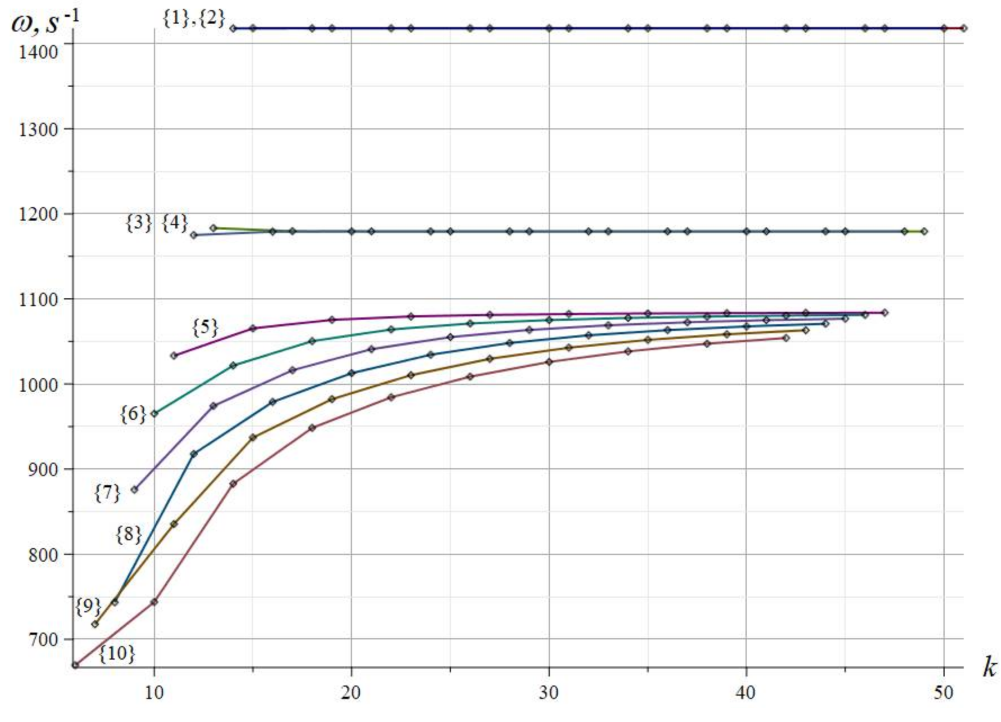

Fig. 7. Isolines of the spectra of regular trusses of various orders

Isolines {5} - {10} asymptotically tend to some constant frequency. Isolines {1}-{4}, starting from some number, are *spectral constants*. The spectra of spatial regular trusses were studied in [23].

Conclusion

The considered truss has one very important property. All rods in it have the same length. In fact, this is a two parametric system. One parameter is the length of the rod, the other is the number of panels n in half the length of the crossbar. Despite this, the obtained analytical solutions are not much simpler than solutions for systems with three or more parameters. A similar composite truss without taking into account the compliance of supports and with an arbitrary number not only in the crossbar, but also in the side parts, was studied in [14, p. 84]

Analytical estimates of deflection and vibration frequencies for practical engineers do not require additional computer calculations and can serve as an approximate solution for designed structures and for checking the accuracy of numerical solutions. A distinctive feature of the Dunkerley frequency estimate in this work is its high accuracy. In spatial systems [23], such an accuracy cannot be obtained. As a rule, the accuracy of the lower estimate of the first frequency of natural oscillations of spatial systems [23] is about 50%, which forces us to use the upper Rayleigh estimate in such problems, which has a more cumbersome form.

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ДЕФОРМАЦИИ И СПЕКТР СОБСТВЕННЫХ ЧАСТОТ ПЛОСКОЙ РЕГУЛЯРНОЙ ФЕРМЫ С ТРЕУГОЛЬНОЙ РЕШЕТКОЙ

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Формула зависимости прогиба статически определимой фермы рамного типа от числа панелей выводится методом индукции в системе компьютерной математики Maple. Рассмотрена равномерная и сосредоточенная нагрузка на верхний пояс. Приведена картина распределения усилий по стержням фермы. Методом Донкерлея найдена аналитическая оценка снизу первой частоты колебаний в предположении, что масса фермы равномерно распределена по узлам. Обобщение решения на произвольное число панелей производится методом индукции. Решение сравнивается с численным решением для всего спектра частот, найденного при помощи оператора решения задачи о собственных числах Eigenvalues системы Maple. Отмечается высокая точность полученной оценки, растущая с увеличением числа панелей. В множестве спектров регулярных ферм различного порядка обнаружены спектральные изолинии и спектральные константы. Погрешность аналитической оценки, представляющей формулу с коэффициентами в виде полиномов по числу панелей не выше шестого порядка, не превышает 24%. При числе панелей больше восьми погрешность оценки меньше 5%.

Ключевые слова: ферма, рама, метод Донкерлея, колебания, основная частота, индукция, Maple, изолинии, спектральные константы