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Ильин И.А., Кирсанов М.Н. Прогиб и смещение опоры фермы с ромбической решеткой

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The deflection and displacement of the bearings
of the truss with rhombic lattice

Решетка балочной статически определимой фермы с двумя боковыми горизонтальными и двумя вертикальными опорными стержнями имеет шпренгельную форму с вертикальными стойками. Методом индукции получены формулы для деформаций фермы и усилий в экстремально сжатых и растянутых стержнях при произвольном числе панелей. Обнаружена и доказана кинематическая изменяемость фермы с четным числом панелей

Ключевые слова: ферма, прогиб, формула Максвелла-Мора, Maple

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Lattice of the beam statically determinate truss with two side horizontal and the two vertical support rods has the vertical struts. By induction the formulas for deformations of the truss and efforts in extremely compressed and stretched rods for an arbitrary number of panels are obtained. The case of kinematic variability of the truss with even number of panels is discovered and proved

Key words: truss, deflection, Maxwell-Mohr' formula, Maple

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Truss (Fig. 1) is externally statically indeterminate. For finding the reactions of its four pillars it is necessary to make the equations of equilibrium of all its nodes [1, 2]. If a small number of panels it is possible to perform, but in the General case the solution of this problem it is time-consuming.

Induction method, developed in [3-12] for the analysis of the deflection of flat trusses and in [13-14] spatial ones allows to obtain analytical solution to such problems, regardless of their external redundancies. Let us apply the algorithm [15] of calculating the stresses in the bars of the truss to find the displacement of the support and deflection of the truss.

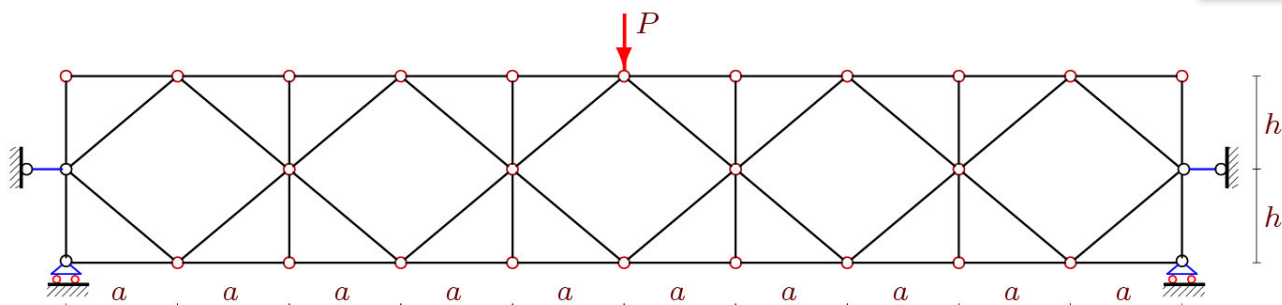


Fig. 1. Truss, $k=3, n=2k-1=5$

The corresponding program is written in the language of symbolic mathematics Maple. Solution begin by entering the coordinates of the nodes. We give the fragment program for this truss:

```
> for i to 2*n+1 do
x[i]:=a*i-a: y[i]:=0:
x[i+2*n+1]:=a*i-a: y[i+2*n+1]:=2*h:
od:
> for i to n+1 do
x[i+4*n+2]:=2*a*i-2*a: y[i+4*n+2]:=h:
> od:
```

The order of connection of rods and nodes is set by special vectors with the numbers of the member ends. For belts these vectors have the form

```
> for i to 2*n do
> N[i]:=[i,i+1];N[i+2*n]:=[i+2*n+1,i+2*n+2];
> od:
```

During computing the efforts it is shown that with an even number of panels determinant of the system is zero, which indicates the kinematic variability of the truss. When $n=2$, the scheme of possible speeds is shown in Fig. 2.

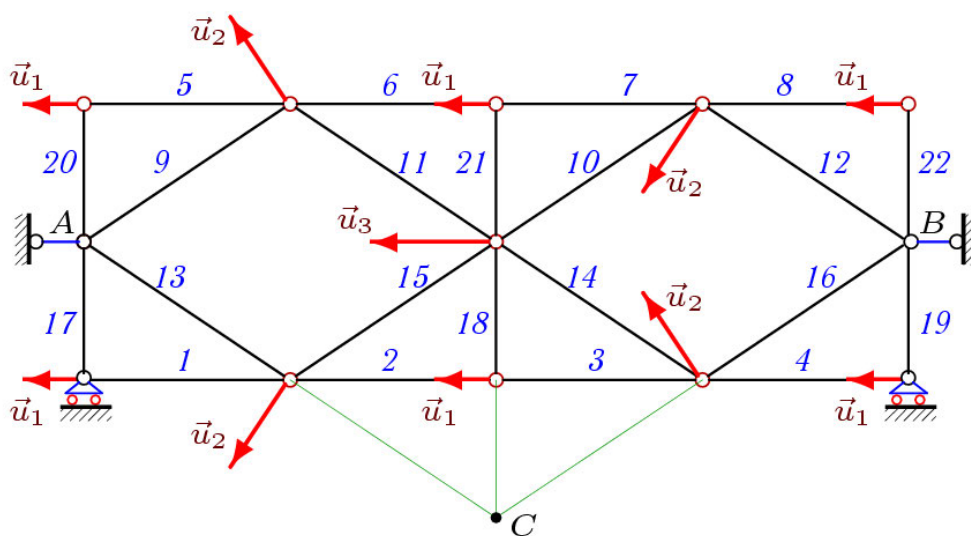


Fig. 2. The variable truss $n=2$. Scheme of possible speeds

The deflection calculated by the formula of Maxwell – Mohr $\Delta = \sum_{i=1}^{m-4} S_i s_i l_i / (EF)$, where S_i – the forces in the rods of the truss from the action of external loads, s_i – stress from the action of a unit horizontal load applied to the movable support, l_i – the lengths of the rods EF – their stiffness, $m=10n+6$, the number of rods. Four anchor rod is assumed to be rigid. Consistent calculation of truss 1, 2, 3 ... 10 panels shows that the formula for the offset is one and the same form $EF\Delta = PA_k a^2 / h$, where the coefficient

$$A_k = (2(-1)^{k+1}k - 2k^2 + (-1)^k + 2k - 1) / 4.$$

Similarly, we can find the deflection of the truss from the action of a uniform load on the bottom flange (Fig. 3): $\Delta = P \frac{A_k a^3 + H_k h^3 + C_k c^3}{2EFh^2}$, where $c = \sqrt{a^2 + h^2}$ and the coefficients have the form $A_n = 2k(5k^3 - 10k^2 + 7k - 2) / 3$, $H_n = 4k - 3$, $C_n = 2k(k - 1)$.

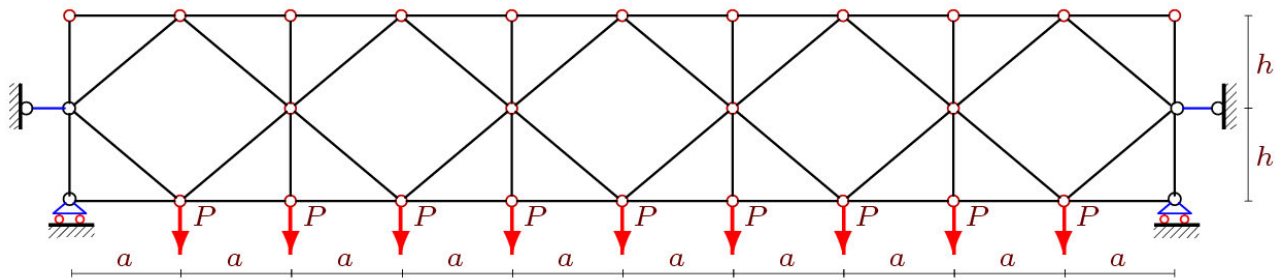


Fig. 3. Uniform load

To obtain a coefficient A_n it is required to find the General term of the sequence 0, 16, 128, 496, 1360, 3040, 5936, 10528, 17376, 27120. To do this, operators of Maple were received and solved the recurrence equation of the fifth order $A_k = 5A_{k-1} - 10A_{k-2} + 10A_{k-3} - 5A_{k-4} + A_{k-5}$. For other coefficients, the equation was simpler: $H_k = 2H_{k-1} - H_{k-2}$, $C_k = 3C_{k-1} - 3C_{k-2} + C_{k-3}$.

At the same time you can find reactions of supports from the action of a uniform load. The efforts in the vertical supports, obtained by the method of induction have the form $R_v = (4k - 3)P / 2$. Efforts in the horizontal (lateral) hinges does not depend on the number of panels: $R_H = Pa / (2h)$.

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